

# PERFORMANCE MODELING OF FMS WITH FLEXIBLE PROCESS PLANS - A PETRI NET APPROACH

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## Abstract

This paper presents modeling and performance analysis of FMS when flexible process plans for each part type are available using Petri nets (PN) modeling approach. PN have been applied successfully for modeling of discrete event dynamic systems such as FMS that are characterized by conflicts, concurrency, synchronization and deadlocks. An example FMS consisting of four machines with individual input and output buffer has been taken into consideration and its PN model construction is explained. System performance is evaluated in two manufacturing environments (i.e. virtual batch manufacturing and virtual line manufacturing). The analysis will assist the planner in selecting optimum set of operating parameters (such as dispatching rule, number of pallets released to the system etc.) for a given production order to achieve the desired performance measure. Several performance measures such as makespan, mean flow time, maximum flow time and variance of flow time have been used to evaluate system performance.

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**Key Words:** Petri Nets, Simulation, Modelling, FMS, Flexible Process Plans

## 1. INTRODUCTION

FMS can be defined as an automated manufacturing system consisting of multi-functional machines that are interconnected by a material handling system. These systems are designed to combine the efficiency of mass-production line with the flexibility of a job shop to best suit the batch production of mid-volume and mid-variety of products. This is partly due to the fact that flexibility is required by manufacturing companies to stay in a highly competitive and changing business environment. The flexibility of a FMS is mainly due to the capability of performing different operations within the same station and the material handling stations, which provide fast and flexible transfer of parts within the system.

In FMS, flexible process plans (FPPs) are common due to the presence of versatile machines and can adapt to the environment and improve system performance [1]. In general, every machined part may have flexible process plans that are feasible given the available manufacturing resources. Flexible process plans can be generated from the considerations of operation flexibility (possibility of performing an operation on more than a machine), sequencing flexibility (possibility of interchanging the sequence in which required manufacturing operations are performed) and processing flexibility (possibility of producing the same manufacturing feature with alternative operations, or sequence of operations). For implementing FPPs in a manufacturing system, although it does not require either flexible automation system or a computer in theory but practical considerations usually dictate the use of both [2].

The capital-intensive nature of FMS, coupled with their immense strategic potential demands careful attention to decision regarding their design and operation. Design decisions include the specification of system requirements, selection of resources and their configuration. Operational decisions include planning, scheduling and real time control of system operation. These decisions require use of system model that represents the relevant system features. Analysis of model reveals information regarding system behavior and serves as input to decision-making process.

A variety of modeling approaches such as mathematical programming approach [3, 4, 5], markov chain approach [6, 7], queuing networks [8, 9, 10], and computer simulation [11, 12, 13, 14, 15, 16] exist for design and operational analysis of manufacturing system. While these models provided insight into system behavior, they introduced many restrictive assumptions and tended to be computationally complex, making it difficult to model and evaluate manufacturing system in dynamic situation [17]. Petri nets (PN) modeling approach has been successfully employed to model, control and analyze the discrete event dynamic system that are characterized by concurrency or parallelism, asynchronous process, deadlocks, conflicts and event driven process and thus suitable for modeling, control and performance analysis (it provides a means of determining system characteristics) of FMS [18, 19, 20, 21]. However, it has been reported in literature that none of the present PN models incorporate variable process sequencing and a few incorporate variable routing [22]. PN is a graphical and mathematical modeling tool. As a graphical tool, it serves as a visual modeling technique and as a communication aid for describing models. As a mathematical tool, it can be used to set up state equations, algebraic equations, simulations and other mathematical models. PN models FMS in terms of *places*, which represent the state of resources or parts in the system and *transitions*, which represent events or activities in the system. PN models can be analyzed either by invariant analysis or simulation. Invariant analysis is a formal method of analyzing system properties such as liveness and boundedness while simulation analyzes system model by generating actual behavior of the FMS. From operational decisions viewpoint, PN is having following advantages over other modeling approaches (such as markov chains and computer simulation etc) [22]:

- (i) PN can represent a large number of system states in a concise manner.
- (ii) PN can capture precedence relations and structural interactions of stochastic, concurrent and asynchronous events.
- (iii) PN can model several real life system features such as deadlock, conflicts and buffer sizes.
- (iv) PN have an underlying mathematical foundation as it can be used to setup state equations and algebraic equations.
- (v) PN can be directly converted into simulation models.
- (vi) PN are derived from the logical sequencing of the system events and are graphical. Thus, they are easy to understand and communicate to others.

Informally, PN consists of four primitive elements (place, transition, arc and token) (Fig. 1) and the rules that govern their operation. *Places*, pictured as circles, are used to represent system states (availability of resources, operations, and processes). *Transitions*, pictured as bars or rectangles, are used to model the events (start or termination of operation). *Arc* connects places to transitions and transitions to places and direction of the path is indicated by an arrowhead at the end of arc. Thus, directed arcs represent input and output relationships between places and transitions. *Tokens* are conceptual entities and they appear as small solid dots in places and model the objects that move in a real network. Tokens in places and their flow regulated by the execution (firing) of transitions add the dynamics to the PN. Each transition has some number of input and output places. These input and output places

represent the pre- and post-conditions of an event (transition), or the resources required and released by the execution of an event. Formally, a five tupled PN is defined as given below.

$PN = (P, T, I, O, M_0)$ ; where:

$P = \{p_1, p_2, \dots, p_m\}$  is a finite set of places,  $m \geq 0$ .

$T = \{t_1, t_2, \dots, t_n\}$  is a finite set of transitions,  $n \geq 0$ ,  $P \cup T \neq \phi$  and  $P \cap T = \phi$ .

$I : (P \times T) \rightarrow N$  is an input function that defines directed arcs from places to transitions, where  $N$  is a set of non-negative integer i.e.  $N = \{0, 1, 2, \dots\}$ .

$O : (T \times P) \rightarrow N$  is an output function which defines directed arcs from transitions to places.

$M_0: P \rightarrow N$  is the initial marking. It is a  $| P |$  dimensional vector with  $M_0(p)$  being the initial token count of a place  $p$ .

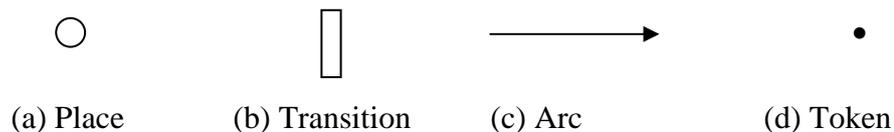


Figure 1: Description of Petri Net Primitives.

A Petri net structure without a specific initial marking is denoted by  $Z$ . A PN structure with a given initial marking is denoted by  $(Z, M_0)$ . A given marking  $M_i$  denotes the current state of a PN.

A PN graph with marking is the dynamic representation of a system with assignment of tokens to the places of the system. The distribution of tokens in a marked PN defines the status of the net. The marking of a PN is contained in a vector of dimension  $m$ , where  $m$  is the number of places and each value of vector corresponds to the number of token in the corresponding places. A token at a place means that corresponding condition holds. When there is a token in each of the input place of a transition, the transition is enabled. An enabled transition can fire and this represents the occurrence of an event or start of a new activity in the system. Thus, transition firing is equivalent to a state change and describes the PN's dynamic behavior. Firing of a transition changes the marking of the PN. A transition fires by removing a token from each of its input place and by placing a token in each of its output places. The firing of a transition causes tokens to flow through the net. Readers can refer to [23, 24, 25, 26, 27] for theories and applications of PN.

This paper presents the PN model of a FMS in which part types with flexible process plans can be manufactured. The PN model construction is explained in detail and the performance analysis of the FMS is carried out in two operating environments viz. VBM and VLM. VBM (virtual batch manufacturing) represents the manufacturing scenario in which operation times are higher and not comparable with transportation times while VLM (virtual line manufacturing) represents the manufacturing scenario in which operation times are comparable with transportation times.

## **2. FMS DESCRIPTION AND ITS WORKING**

An example FMS (Fig. 2) consisting of four CNC machines (M1, M2, M3, and M4), each with an input and output buffer of capacity three and two respectively is taken into consideration. A giant robot R-2 is available to serve all four machines. There is a load and unload (L/UL) station of infinite capacity to load/unload the parts on/from the pallets. The system input (B-IN) and output buffer (B-OUT) have a capacity of ten and eight respectively. There is another robot R-1 in the system, which operates between L/UL station and B-IN/B-OUT for part transfer. Travel times of R-1 and R-2 while serving other system resources are shown in Table I.

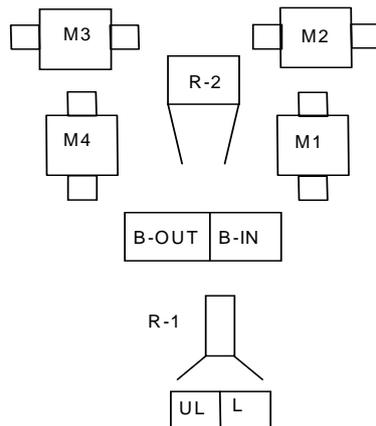


Figure 2: Taken FMS Configuration [28].

The part flows into the FMS is as follows: A piece of raw material is first loaded onto a pallet at load station by manual operation. The palletized part is then transported to B-IN by R-1. R-2 then moves parts from B-IN to various machines according to their process plans. When FPPs are available, a part can switch over to alternate process plan to overcome the non-availability of machine due to limited buffer space. The sequence of part flow at a machine is: machine input buffer → machine table → machine output buffer. When all the machining operations on a part are completed, R-2 transports it back to B-OUT. R-1 again transports finished parts from B-OUT to unload station, where part is unloaded from the pallet manually. This empty pallet is again loaded with raw part of same part type, if available, and sent into the system. At any time, three job types viz., raw jobs, semi-finished jobs and finished jobs are circulating and compete for the same resources (machines/robots). Thus, several operational decisions such as part selection decision (required at B-IN, B-OUT, input and output buffer of each machine) and machine selection decision (required at B-IN and output buffer of each machine in case of FPPs) are required for the smooth functioning of the FMS. These decisions are not described here for want of space and discussed in [28].

### 3. PETRI NET MODELING OF FMS

The PN construct of the example FMS system (Fig. 2) is depicted in Fig. 3. Events occurring in the manufacturing system are represented by timeless (immediate) transitions that are depicted by single bars. Similarly activities performed for the part manufacturing such as part processing, transfer of a part by robot are represented by timed transitions that are depicted by rectangles. System states such as availability of resources and parts are represented by places, which are pictured, as circles. Directed arcs that represent input and output relationships between places and transitions as well as tokens that appear as small solid dots in places and model the objects that move in a real network are also shown. Description of various immediate as well as timed transitions and places of the PN construct are given in tables II, III, IV respectively.

The construction of PN model can be understood by analyzing the flow of a job through the FMS. Initially part specific pallet, raw job and robot R-1 are available. It is assumed that raw jobs are loaded onto the pallets manually. An activity begins when R-1 picks the palletized part (if there is vacancy in system input buffer (B-IN)) and transports it to B-IN. After palletized part reaches in B-IN, R-1 becomes free and available again for new assignment. Availability of part specific pallet, raw job and R-1 is shown by places  $p_{59}$ ,  $p_1$  and  $p_2$  respectively.

Table I: R-1 and R-2 Unit Travel Times [28].

		L/ UL	B-IN/ B-OUT	M1	M2	M3	M4
Robot R-1	L/UL	0	2	-	-	-	-
	B-IN/ B-OUT	2	0	-	-	-	-
Robot R-2	B-IN/ B-OUT	-	0	3	5	5	3
	M1	-	3	0	3	5	7
	M2	-	5	3	0	3	5
	M3	-	5	5	3	0	3
	M4	-	3	7	5	3	0

Legend: – Not Applicable.

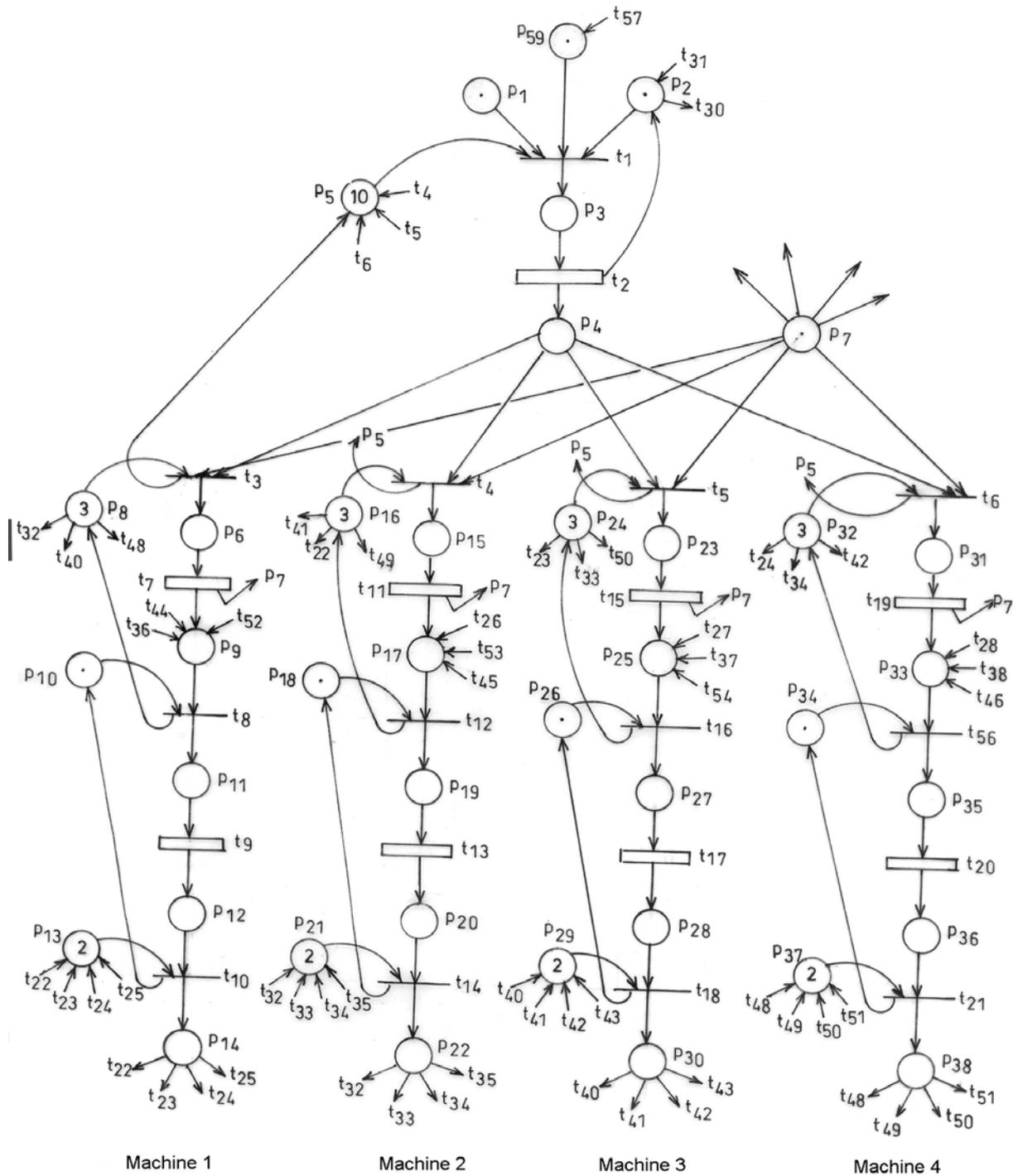


Figure 3a: Petri Net Model of FMS.

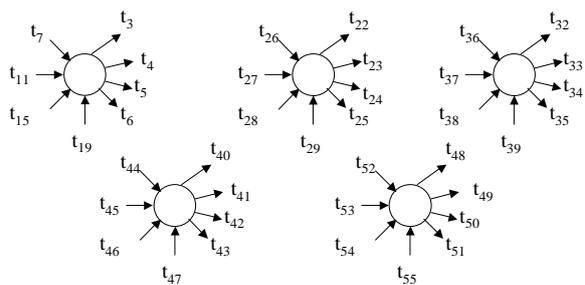


Figure 3b: Interaction from and to P7.



Table III: Description of Timed Transitions of System PN Model.

Timed Transitions	Description
$t_2$	Part transfer by R-1
$t_7, t_{11}, t_{15}, t_{19}, t_{26}, t_{27}, t_{28}, t_{29}, t_{31}$	Part transfer by R-2
$t_9$	Processing by machine 1
$t_{13}$	Processing by machine 2
$t_{17}$	Processing by machine 3
$t_{20}$	Processing by machine 4
$t_{36}, t_{37}, t_{38}, t_{39}, t_{44}, t_{45}, t_{46}, t_{47}, t_{52}, t_{53}, t_{54}, t_{55}$	Part transfer by R-2

Table IV: Description of Places of System PN Model.

Places	Description
$P_1$	Parts Available
$P_2$	Robot 1 (R-1) Available
$P_3$	R-1 transporting a part
$P_4$	Part is in B-IN ( Buffer capacity = 10)
$P_5$	Vacancy in B-IN
$P_6, P_{15}, P_{23}, P_{31}, P_{39}, P_{40}, P_{41}, P_{43}$	R-2 transporting a part
$P_7$	R-2 available
$P_8$	Vacancy in input buffer of machine 1
$P_9$	Part is in input buffer of machine 1 ( Buffer capacity = 3)
$P_{10}$	Machine 1 available
$P_{11}$	Machine 1 processing a part
$P_{12}$	A part that has just been processed by machine 1
$P_{13}$	Vacancy in output buffer of machine 1
$P_{14}$	Part is in output buffer of machine 1 ( Buffer capacity = 2)
$P_{16}$	Vacancy in input buffer of machine 2
$P_{17}$	Part is in input buffer of machine 2 ( Buffer capacity = 3)
$P_{18}$	Machine 2 available
$P_{19}$	Machine 2 processing a part
$P_{20}$	A part that has just been processed by machine 2
$P_{21}$	Vacancy in output buffer of machine 2
$P_{22}$	Part is in output buffer of machine 2 ( Buffer capacity = 2)
$P_{24}$	Vacancy in input buffer of machine 3
$P_{25}$	Part is in input buffer of machine 3 ( Buffer capacity = 3)
$P_{26}$	Machine 3 available
$P_{27}$	Machine 3 processing a part
$P_{28}$	A part that has just been processed by machine 3
$P_{29}$	Vacancy in output buffer of machine 3
$P_{30}$	Part is in output buffer of machine 3 ( Buffer capacity = 2)
$P_{32}$	Vacancy in input buffer of machine 4
$P_{33}$	Part is in input buffer of machine 4 ( Buffer capacity = 3)
$P_{34}$	Machine 4 available
$P_{35}$	Machine 4 processing a part
$P_{36}$	A part that has just been processed by machine 4
$P_{37}$	Vacancy in output buffer of machine 4
$P_{38}$	Part is in output buffer of machine 4 ( Buffer capacity = 2)
$P_{42}$	Vacancy in B-OUT
$P_{44}$	Part is in B-OUT ( Buffer capacity = 8)
$P_{45}, P_{46}, P_{47}, P_{48}, P_{49}, P_{50}$	R-2 transporting a part
$P_{51}, P_{52}, P_{53}, P_{54}, P_{55}, P_{56}, P_{57}$	R-2 transporting a part
$P_{58}$	Part is at unload station
$P_{59}$	Pallets available

Immediate transition  $t_1$  represents the start of transportation activity and timed transition  $t_2$  represents the time to transport the job from load station to B-IN. Place  $p_5$  represents the availability of space in B-IN.

After the part reaches in B-IN, it can go to any of machine 1 (M1), machine 2 (M2), machine 3 (M3) or machine 4 (M4) according to the followed process plan and the availability of the required machine as flexible process plans for each part type are available and a part type can change its process plan to overcome the uncertainty occurring on the shop floor such as non-availability of machine. The machine selection at this stage can be performed by applying a machine selection rule [28]. Robot R-2 is then assigned to transport the part from B-IN to input buffer of the selected machine. After palletized part reaches in the input buffer of the machine, R-2 becomes free and available again. Immediate transitions  $t_3$ ,  $t_4$ ,  $t_5$  and  $t_6$  represent the start of transportation activity by R-2 to M1, M2, M3 and M4 respectively. Similarly, timed transitions  $t_7$ ,  $t_{11}$ ,  $t_{15}$  and  $t_{19}$  represent the time of transportation from B-IN to input buffer of M1, M2, M3 and M4 respectively by R-2. Places  $p_8$ ,  $p_{16}$ ,  $p_{24}$  and  $p_{32}$  represent availability of space in the input buffer of M1, M2, M3 and M4 respectively.

Once part reaches in input buffer of the machine, it will be loaded onto machine automatically by shuttle mechanism (if machine is available) and machine starts processing of the loaded part. After processing of part is finished, the palletized part is transferred to the output buffer of the machine (if there is a vacancy in it) automatically. Places  $p_{10}$ ,  $p_{18}$ ,  $p_{26}$  and  $p_{34}$  represent the availability of M1, M2, M3 and M4 respectively. Immediate transitions  $t_8$ ,  $t_{12}$ ,  $t_{16}$  and  $t_{56}$  represent the start of the processing of part by the M1, M2, M3 and M4 respectively. Timed transitions  $t_9$ ,  $t_{13}$ ,  $t_{17}$  and  $t_{20}$  represent the time to process the part on M1, M2, M3 and M4 respectively. In the present work, operation times are assumed to be predetermined as well as deterministic and they will be retrieved from the inputted process plan of the respective part. Places  $p_{12}$ ,  $p_{20}$ ,  $p_{28}$  and  $p_{36}$  represent the post operation state of the part that just has been processed by M1, M2, M3 and M4 respectively. Places  $p_{13}$ ,  $p_{21}$ ,  $p_{29}$  and  $p_{37}$  represent the availability of space in the output buffer of M1, M2, M3 and M4 respectively.

From output buffer of the machine, the palletized part can go to any of the remaining machine (if the part has not yet been finished) or to B-OUT (if the part has been finished). Accordingly, robot R-2 then transports the part either to the input buffer of the next required machine or to the B-OUT. Once the part reaches at its destination (a machine or B-OUT), R-2 becomes free and available. From output buffer of M1 ( $p_{14}$ ), start of transportation activity of part by R-2 to M2, M3, M4 and B-OUT are represented by immediate transitions  $t_{22}$ ,  $t_{23}$ ,  $t_{24}$  and  $t_{25}$  respectively. Similarly, immediate transitions  $t_{32}$ ,  $t_{33}$ ,  $t_{34}$  and  $t_{35}$  represent the above-mentioned event from output buffer of M2 ( $p_{22}$ ) to M1, M3, M4 and B-OUT respectively. Immediate transitions  $t_{40}$ ,  $t_{41}$ ,  $t_{42}$  and  $t_{43}$  represent the same event from output buffer of M3 ( $p_{30}$ ) to M1, M2, M4 and B-OUT respectively while immediate transitions  $t_{48}$ ,  $t_{49}$ ,  $t_{50}$  and  $t_{51}$  represent the above mentioned event from the output buffer of M4 (place  $p_{38}$ ) to M1, M2, M3 and B-OUT respectively. Timed transitions  $t_{26}$ ,  $t_{27}$ ,  $t_{28}$ ,  $t_{29}$ ,  $t_{36}$ ,  $t_{38}$ ,  $t_{39}$ ,  $t_{44}$ ,  $t_{45}$ ,  $t_{46}$ ,  $t_{47}$ ,  $t_{52}$ ,  $t_{53}$ ,  $t_{54}$  and  $t_{55}$  represent the time of transportation to input buffer of different machines as shown in table I. Places  $p_{39}$ ,  $p_{40}$ ,  $p_{41}$ ,  $p_{47}$ ,  $p_{48}$ ,  $p_{49}$ ,  $p_{50}$ ,  $p_{51}$ ,  $p_{52}$ ,  $p_{53}$ ,  $p_{54}$ ,  $p_{55}$ ,  $p_{56}$  and  $p_{57}$  represent that R-2 is busy in transporting a part.

After part reaches at B-OUT ( $p_{44}$ ), robot R-1 (if available) will transport the finished part to unload station. Immediate transition  $t_{30}$  represents the start of transportation activity by R-1 from B-OUT to unload station. Timed transition  $t_{31}$  represents the time to transport the part from B-OUT to unload station while place  $p_{45}$  denotes that R-1 is busy in transporting the part. At unload station, finished part is unloaded manually from the pallet, making the part specific pallet free. This empty pallet will again be loaded with raw part of same part type (if there is any remaining part of same part type). Immediate transition  $t_{57}$  represents the

unloading of finished part from pallet while place  $p_{58}$  represents the finished part is at unload station thus completing the PN model of the FMS.

Model validation is important in determining that the developed model is an accurate representation of the real world system under study. Several methods such as coverability tree and incidence matrix are used to validate the PN model [27]. As present work uses a hypothetical FMS, instead of validation, verification of the model is necessary. In the present work, it is carried out in three stages (i) debugging of the C code, (ii) checking the internal logic of the model and (iii) comparing the model output with information gathered from a manual simulation using the same data for four production orders.

#### 4. PERFORMANCE ANALYSIS

Experiments are designed and conducted in two manufacturing environments viz., VBM and VLM. Tables V and VI show the process plans of various part types and variables with their range/values considered in the present study respectively. Number of pallets released to the system (np) varies from ten (50 % of work-in-process) to eighteen (90 % of work-in-process) in steps of two. Four dispatching rules i.e. shortest processing time (SPT), processing time over remaining operation (PT2), percentage of completed work (%CW) and percentage of sum of completed work and operation time (CW1) are used in the present work. It is important to mention that authors proposed four dispatching rules (%CW, CW1, CW2, CW5) in earlier paper [29]. Assumptions such as (i) operations and transportation times are deterministic, (ii) each machine is continuously available for processing jobs, (iii) part preemption is not allowed (iv) pallet and fixture availability is limited and it can load one part at the most and (v) number of pallets available for each part type are in same proportion of the

Table V: Flexible process plans of various part types.

Table VI: Parameters and their ranges.

Part Type Number	VLM $M_i(T_i)$	VBM $M_i(T_i)$	S. No.	Parameter	Range/Value Employed	
1.	1(4) – 3(7) – 1(3) – 4(2)	1(60) – 3(75) – 2(60) – 4(30)	1.	Production quantity of each part type in a production order	20-50	
	1(4) – 3(7) – 2(5) – 4(2)	1(60) – 3(75) – 1(58) – 4(30)				
	2(6) – 3(7) – 1(3) – 4(2)	2(65) – 3(75) – 2(60) – 4(30)		2.	Number of part types in a production order	3-4
	2(6) – 3(7) – 2(5) – 4(2)	2(65) – 3(75) – 1(58) – 4(30)				
2.	2(4) – 1(3) – 2(2) – 4(10)	2(60) – 1(45) – 2(41) – 4(90)		3.	Number of operations per job	4
	2(4) – 1(3) – 2(2) – 1(12)	2(60) – 1(45) – 3(45) – 4(90)				
	2(4) – 1(3) – 3(3) – 4(10)	3(65) – 1(45) – 2(41) – 4(90)		4.	Manufacturing system environment	VBM and VLM
	3(7) – 1(3) – 2(2) – 4(10)	3(65) – 1(45) – 3(45) – 4(90)				
3.	1(5) – 4(2) – 3(2) – 2(3)	1(60) – 4(45) – 3(40) – 2(30)		5.	Transportation time	2-7 units
	1(5) – 3(2) – 4(3) – 2(3)	1(60) – 3(41) – 4(45) – 2(30)				
	1(5) – 2(4) – 3(2) – 2(3)	1(60) – 3(41) – 2(45) – 4(33)	6.	Operation time	2-15 units (for VLM environment) and 20-100 units (for VBM environment)	
	1(5) – 2(4) – 4(3) – 2(3)	1(60) – 4(45) – 2(45) – 1(32)				
4.	2(3) – 1(6) – 2(5) – 1(3)	1(60) – 2(82) – 3(100) – 2(100)	7.	Number of pallets released to the system	10-18 in steps of 2 i.e. 10, 12, 14, 16, 18	
	2(3) – 1(6) – 2(5) – 3(4)	1(60) – 4(80) – 3(100) – 2(100)				
	3(6) – 4(5) – 2(5) – 1(3)	3(65) – 2(82) – 1(98) – 4(100)	8.	Dispatching rule	SPT, PT2, %CW, CW1	
	3(6) – 1(6) – 2(5) – 1(3)	3(65) – 4(80) – 1(98) – 4(100)				
5.	4(4) – 3(4) – 2(4) – 4(8)	4(97) – 3(54) – 2(40) – 1(99)	9.	Performance measure	Makespan, mean flow time, maximum flow time, variance of flow time	
	2(5) – 1(3) – 2(4) – 4(8)	4(97) – 1(50) – 3(42) – 4(100)				
	2(5) – 3(4) – 2(4) – 4(8)	2(100) – 3(54) – 2(40) – 1(99)				
	2(5) – 1(3) – 3(8) – 4(8)	2(100) – 1(50) – 3(42) – 4(100)				
6.	4(6) – 2(3) – 3(2) – 1(3)	2(60) – 1(100) – 2(100) – 3(80)	Legend: VBM – virtual batch manufacturing			
	3(7) – 2(3) – 3(2) – 1(3)	2(60) – 1(100) – 4(97) – 3(80)	VLM – virtual line manufacturing			
	4(6) – 2(3) – 3(2) – 2(4)	3(63) – 4(100) – 2(100) – 1(78)				
	4(6) – 2(3) – 4(2) – 1(3)	3(63) – 4(100) – 2(100) – 3(80)				
7.	3(4) – 4(4) – 3(5) – 4(5)	4(57) – 1(95) – 2(62) – 4(40)				
	1(3) – 2(5) – 3(5) – 2(6)	4(57) – 1(95) – 2(62) – 1(42)				
	1(3) – 4(4) – 3(5) – 4(5)	1(60) – 2(100) – 3(60) – 4(40)				
	3(4) – 2(5) – 3(5) – 4(5)	1(60) – 2(100) – 3(60) – 1(42)				

Legend:  $M_i$  - Machine Number,  $T_i$  - Operation Time

total pallets available as the proportion of their required production quantity with respect to the total production requirement.

Experiments are performed by simulating the PN model and in each manufacturing environment, two case studies (total four case studies) are performed and 80 (= 5 (number of np) × 4 (type of DR) × 4 (number of case studies)) simulation runs are carried out by simulating the PN model of FMS. Results obtained from simulation are depicted in Tables VII-X.

Table VII: Result for Case Study 1 (VBM Environment).

	Performance Measures	Dispatching Rule				
		CW1	PT2	SPT	%CW	
Number of Pallets Released	10	Makespan	<b>13005</b>	16805	15646	13898
		Mean flow time	766.98	844.38	749.13	<b>739.12</b>
		Max. Flow time	<b>1721</b>	2676	10363	3817
		Variance(Flow time)	<b>75015.06</b>	227254.82	1212709.16	287053.08
	12	Makespan	16329	16968	16103	<b>13143</b>
		Mean flow time	1002.44	944.51	<b>905.11</b>	908.17
		Max. Flow time	<b>2767</b>	5561	9942	8134
		Variance(Flow time)	<b>219053.01</b>	531585.81	1450386.26	761010.16
	14	Makespan	14811	14650	14628	<b>13826</b>
		Mean flow time	1142.94	1119.53	<b>1050.15</b>	1236.40
		Max. Flow time	<b>3328</b>	4512	6326	10077
		Variance(Flow time)	<b>206934.11</b>	461110.39	959564.36	1524975.51
16	Makespan	13181	14448	14810	<b>12964</b>	
	Mean flow time	1210.26	<b>1176.19</b>	1210.82	1191.49	
	Max. Flow time	<b>4789</b>	9275	10549	9361	
	Variance(Flow time)	<b>377210.95</b>	936241.20	1287339.60	1636878.59	
18	Makespan	13558	13630	15220	<b>13543</b>	
	Mean flow time	1315.36	<b>1223.15</b>	1360.42	1316.47	
	Max. Flow time	10416	7933	11426	<b>6975</b>	
	Variance(Flow time)	987020.63	<b>943914.90</b>	109462.11	111601.21	

Table VIII: Result for Case Study 2 (VBM Environment).

	Performance Measures	Dispatching Rule				
		CW1	PT2	SPT	%CW	
Number of Pallets Released	10	Makespan	7814	7847	<b>7561</b>	7706
		Mean flow time	564.33	<b>555.23</b>	558.24	576.26
		Max. Flow time	2578	6077	<b>992</b>	1290
		Variance(Flow time)	63230.46	29435.89	<b>15498.14</b>	50423.29
	12	Makespan	<b>7337</b>	8047	8027	8391
		Mean flow time	638.48	620.99	<b>611.73</b>	644.33
		Max. Flow time	2296	2041	<b>1466</b>	2438
		Variance(Flow time)	63011.31	52213.54	<b>30358.12</b>	116645.41
	14	Makespan	7449	7450	7550	<b>7376</b>
		Mean flow time	<b>693.76</b>	715.08	714.54	717.36
		Max. Flow time	3088	2098	<b>1275</b>	2895
		Variance(Flow time)	236790.42	74968.46	<b>33036.50</b>	221924.50
16	Makespan	7705	<b>7199</b>	7438	7521	
	Mean flow time	784.23	836.75	795.67	<b>779.28</b>	
	Max. Flow time	3687	1982	<b>1293</b>	3582	
	Variance(Flow time)	459613.25	70738.26	<b>52989.12</b>	402639.49	
18	Makespan	7868	<b>7213</b>	7464	7685	
	Mean flow time	935.90	972.35	<b>893.29</b>	948.23	
	Max. Flow time	5451	1928	<b>1517</b>	4773	
	Variance(Flow time)	959600.34	93807.12	<b>66535.28</b>	500523.13	

Table IX: Result for Case Study 3 (VLM Environment).

	Performance Measures	Dispatching Rule				
		CW1	PT2	SPT	%CW	
Number of Pallets Released	10	Makespan	2119	2122	2130	<b>2072</b>
		Mean flow time	171.37	<b>151.34</b>	152.37	160.48
		Max. Flow time	<b>312</b>	525	570	361
		Variance(Flow time)	<b>2486.33</b>	9963.19	10233.93	4460.12
	12	Makespan	2081	2077	<b>2067</b>	2095
		Mean flow time	183.46	167.59	<b>167.20</b>	187.24
		Max. Flow time	<b>310</b>	375	375	458
		Variance(Flow time)	<b>2583.00</b>	6903.29	6943.91	6910.51
	14	Makespan	2075	2086	<b>2070</b>	2078
		Mean flow time	232.56	213.67	<b>213.33</b>	231.12
		Max. Flow time	553	566	570	<b>427</b>
		Variance(Flow time)	<b>6864.54</b>	9025.63	10415.44	5620.05
	16	Makespan	2075	2080	2073	<b>2071</b>
		Mean flow time	230.57	227.32	<b>227.23</b>	258.63
		Max. Flow time	1548	1217	1211	<b>555</b>
		Variance(Flow time)	49126.01	31138.13	33575.87	<b>7917.39</b>
18	Makespan	2075	<b>2070</b>	2075	<b>2070</b>	
	Mean flow time	266.64	<b>260.12</b>	260.47	294.49	
	Max. Flow time	1624	1454	1445	<b>585</b>	
	Variance(Flow time)	68422.74	55013.88	55028.07	<b>7515.02</b>	

Table X: Result for Case Study 4 (VLM Environment).

	Performance Measures	Dispatching Rule				
		CW1	PT2	SPT	%CW	
Number of Pallets Released	10	Makespan	2461	2450	<b>2441</b>	2456
		Mean flow time	<b>169.32</b>	169.87	183.12	169.98
		Max. Flow time	1872	1861	<b>435</b>	1842
		Variance(Flow time)	53644.81	53334.77	<b>5729.94</b>	50739.03
	12	Makespan	2423	2421	2420	<b>2419</b>
		Mean flow time	<b>210.77</b>	213.29	216.84	217.23
		Max. Flow time	888	613	<b>592</b>	629
		Variance(Flow time)	17268.10	12261.13	<b>7882.90</b>	11971.14
	14	Makespan	2421	2421	<b>2418</b>	2419
		Mean flow time	248.11	<b>242.72</b>	259.19	282.23
		Max. Flow time	1928	1919	<b>729</b>	1344
		Variance(Flow time)	88278.36	91085.35	18986.28	<b>17467.54</b>
	16	Makespan	2421	2421	2421	<b>2418</b>
		Mean flow time	<b>271.14</b>	278.17	272.63	308.12
		Max. Flow time	1953	1959	1909	<b>615</b>
		Variance(Flow time)	102574.38	103883.02	96573.17	<b>10504.12</b>
18	Makespan	2421	2421	<b>2418</b>	2421	
	Mean flow time	<b>296.77</b>	305.11	305.27	334.25	
	Max. Flow time	1952	1953	<b>1938</b>	7898	
	Variance(Flow time)	107208.12	104205.31	<b>10250.42</b>	19782.44	

Tables VII-X clearly reveals that for different system parameters such as number of pallets released to the system and dispatching rule, the system performance as measured by makespan, mean flow time, maximum flow time and variance of flow time is different. Table XI shows the set of best operating policies for taken case studies. For Case Study 1 and to have minimum makespan, the number of pallets released to the system should be sixteen with %CW dispatching rule. Similarly, for minimum mean flow time, the number of pallets released to the system should be ten with %CW dispatching rule, for minimum of maximum flow time performance measure, the number of pallets released to the system is ten with CW1 dispatching rule and for minimum of variance of flow time, the set of operating policies should be CW1 dispatching rule with ten number of pallets released to the system. Similarly, for other case studies the best operating policies can be known from Table XI.

Table XI: Best Performing Dispatching Rule and Number of Pallets.

S. No.	Performance Measure	Case Study 1		Case Study 2		Case Study 3		Case Study 4	
		np	DR	np	DR	np	DR	np	DR
1	Makespan	16	%CW	16	PT2	12	SPT	14, 16, 18	SPT, %CW, SPT
2	Mean Flow Time	10	%CW	10	PT2	10	PT2	10	CW1
3	Maximum Flow Time	10	CW1	10	SPT	12	CW1	10	SPT
4	Variance of Flow Time	10	CW1	10	SPT	10	CW1	10	SPT

Legend: np – number of pallets released to the system, DR – Dispatching rule

## 5. CONCLUSIONS

This paper presents the PN model of a FMS when flexible process plans for each part type are known. PN are the most powerful tool and have been successfully employed to model FMS as they capture various characteristics of FMS such as concurrency, synchronization, conflicts and deadlocks. A hypothetical FMS has been modeled using PN approach and construction has been explained in detail. The PN model is simulated under variety of operating conditions to analyze system performance. The analysis will assist the planner in selecting optimum set of operating parameters such as dispatching rule, number of pallets released to the system for a given production order to achieve the desired performance measure and also to reorganize and re-evaluate manufacturing system so they may respond flexibly.

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