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SIMULATION AND MODELLING OF THE VIBRATION EFFECT ON THE HOLE FORM IN DRILLING

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Abstract
The vibration phenomenon represents the cause of many problems in metal cutting processes. It has a big influence on the quality of the machining operation, tool life and machine precision. Drilling operation is a common metal cutting process where quality of the hole form is often a critical issue. Therefore, it was necessary to predict these vibrations.
The first part of this paper deals with modelling and writing equation of motion of a two freedom degrees system for drilling operation. Then, in the second part, we develop a numerical resolution of the governing equation (nonlinear and delayed differential equation) using Newmark Method combined with Newton-Raphson Method.
Numerical results of this study have also been shown and comment upon: thrust force and torque (parametrical study in static way), dynamical responses, dynamical cutting force, hole form simulation with amplified vibration amplitude (amplification 1000 time).
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Key Words: Drilling Vibrations, Simulation, Vibratory Behaviour, Hole Form Prediction

1. INTRODUCTION

Hole quality parameters that are generally used to evaluate the quality of a drilled hole are determined from the hole profile. The hole profile, in turn, depends on transverse vibrations of the drill and associated radial forces. A thorough understanding of the factors affecting the hole quality, and eventually the prediction of the hole quality, can only be achieved through an understanding of influence of all factors on the drill dynamics and drilling forces.

Drilling is a fundamental machining process. From the metal cutting point of view, drilling is a very severe process in which the chip load is large and the chips may be trapped in the cutting zone causing various problems [2]. Consider the three most fundamental chip-making processes: milling, drilling and turning. One of these processes stands out. In milling and turning, cutting takes place in the open, making it possible to learn a great deal about the behaviour of the cut through observation alone. But in drilling the action is hidden. No one can see what happens in the cut, and no unaided eye can see the machined surface unless the hole is cut open. This invisibility, as much as any other factor, probably accounts for why the mechanics of drilling have been so poorly understood.

One particularly important missing piece has been an applied understanding of self-excited or "regenerative" vibrations in drilling. These are vibrations such as chatter that feed themselves simply as a result of the dynamics of the cut. The study of drilling has often presented some difficulties which are linked to the complex geometry of the twist drill [1-4]. In practice, generally empirical equations are used to calculate thrust force and torque [2-4]. These equations are very approximate, because they do not take all the cutting parameters into account. They often use only the feed speed and the diameter of the drill [5-7].In this paper, we presented a numerical resolution of the motion equation of a drilling dynamical model. It was two degrees of freedom model. We used also a theoretical model for the calculation of the cutting forces.

Table I: Nomenclature.

f	Feed speed[mm/rev]
i	inclination angle ($i = \arcsin(\sin\beta \sin p)$)
r	Section radius from the drill axis
ψ'	$\psi' = \pi - \psi$ the web angle
p	Half of the drill point angle (degrees)
$2t$	lip spacing
k_{AB}	The shear flow stress along the shear plane.
β	Web angle $\beta = \arcsin\frac{t}{r}$
d	Drill diameter
d'	Chisel edge diameter
ξ	Intermediate angle : $\xi = \arctan(\tan\beta \cos p)$
λ_n	Friction angle on the cutting lips
λ_d	Friction angle on the chisel edge
ϕ_d	Shear angle on the chisel edge
ϕ_n	Shear angle on the cutting lips
γ_n	Cutting angle on the cutting lips
γ_d	Cutting angle on the chisel edge
δ	Helix angle (degrees)
V_C	Cutting speed
N	Spindle speed

2. DYNAMICAL STUDY

2.1 Dynamical model

In the study of the machining process, modelling is very important as it helps us to understand the process and hence, to solve practical problems such as chatter vibrations.

The vibratory behaviour of the tool system depends on the characteristics of its mass–damper–spring system and the type of external excitation, which originates from the cutting process [2]. The cutting forces are influenced by geometrical, dynamic, material and tool surface properties. Different models have been developed [2, 8-10], based on the requirement of simplicity or complexity. In the study of the cutting materials and precisely drilling, the judicious choice of dynamic model is very important to understand and solve the chatter vibration problems.

In the normal drilling, the cutting forces in *x* and *y* directions are equal to zero. However, if there is a vibration, then there will be residual forces in the *x* and *y* directions. These forces may cause the machine tool to vibrate, which in turn causes undulation of the cutting forces. Such a regenerative effect has been studied extensively in turning and milling [11]. However, for drilling operations, it is much more difficult and complex. In fact, in drilling operations,

the interactions between the hole and the tool are rather complicated. In order to simplify the model, a number of assumptions are made [2]:
1) The work piece is rigid so that its vibration is negligible.
2) The tool is rigid in the *z* direction. In other words, the vibrations occur only in the radian direction.
3) The effect of the chip is small and hence, can be ignored.
4) The body of the tool will not touch the hole even under severe vibration. It is known that under normal conditions, the body clearness of the drill prevents the body of the drill from touching the hole. When vibrations occur, the hole is enlarged by the cutting edges of the vibrating tool. Hence, the body of the drill will still not touch the hole.

Fig.1 shows the dynamical model of the machine tool.

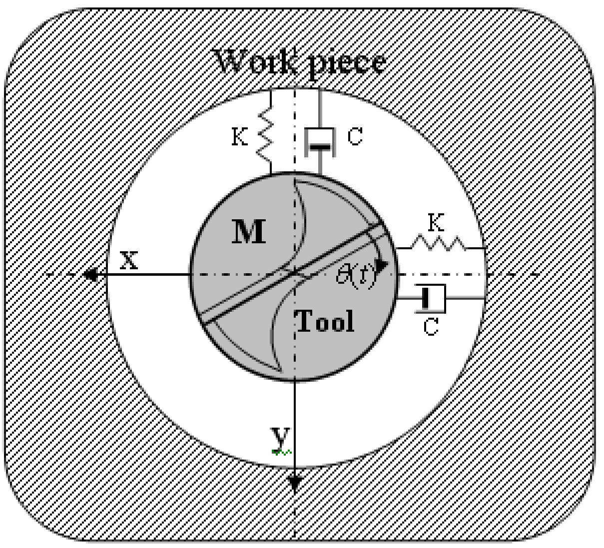


Figure 1: Dynamical drilling model.

2.2 Motion Equations

In the dynamical model, the machine tool is assumed to be rigid in *z* direction and approximated by a set of second order differential equations in *x* and *y* directions:

$$\begin{aligned} M\ddot{x}(t) + C\dot{x}(t) + Kx(t) &= \Delta F_x(x(t), y(t)) \\ M\ddot{y}(t) + C\dot{y}(t) + Ky(t) &= \Delta F_y(x(t), y(t)) \end{aligned} \tag{1}$$

where, *M*, *C*, and *K* are the mass, damping, and stiffness of the tool respectively. It's important to note that the vibration acts on all the cutting edges and has a direct influence on the generated surface of the hole. The vibration will affect the feed speed, so the choice of the forces model can made a big influence on the simulation results.

ΔF_x *and* ΔF_y, represents the residual forces generated by the cutting operation in *x* and *y* directions. Each of them depends at a time of the instantaneous displacement *x* and *y* and the time *t*. In this model, the problem is nonlinear and coupled between *x* and *y* directions. ΔF_x *and* ΔF_y, was deducted from the variations of the torque *C*. These forces are given by the following expressions:

$$\Delta F_x = \frac{\Delta C}{d}\sin\theta, \quad \Delta F_y = \frac{\Delta C}{d}\cos\theta \tag{2}$$

The torque is proportional to the feed speed *f* [1], in fact:

$$C_C = \mathrm{I}C_C \times \Delta f,\ \ C_l = \mathrm{I}C_l \times \Delta f \tag{3}$$

$\mathrm{I}C_l$ *and* $\mathrm{I}C_C$ are the integral parts of the torque on the lips and the chisel edge, respectively. They can be written as follow [1]:

$$\mathrm{I}C_C = 2\int_0^{r_0} \frac{\cos(\phi_d - \gamma_d)}{\cos(\phi_d + \lambda_d - \gamma_d)} \frac{k_{AB}\cos\gamma_f}{2\sin\phi_d} (\sin\gamma_f - \tan(\phi_d - \gamma_d)\sin\gamma_f)dr \tag{4}$$

$$\mathrm{I}C_l = 2\int_{r_0}^{r_1} k_{AB} \frac{\sin p \cos\xi}{2\sin\phi_n \cos\phi_n} (\cos(\phi_n - \gamma_n - i)\sin p - \cos p) \frac{r^2}{(r^2 - t^2)^{1/2}} dr \tag{5}$$

$$\Delta C = (\mathrm{I}C_\mathrm{l} + \mathrm{I}C_\mathrm{C}).\Delta f \tag{6}$$

with the vibration, the chip thickness will change. The chip thickness *h* becomes:

$$h = (\frac{f}{2}\tan p)d\theta \tag{7}$$

The instantaneous variation of the chip thickness is function of the vibration of the tool in the *x* and *y* directions and there to the instant '*t*' and to the delayed instant '*t*-τ/2', [2]:

$$\Delta h = [(x(t) - x(t - \tau/2))\cos\theta + (y(t) - y(t - \tau/2))\sin\theta]d\theta \tag{8}$$

This instantaneous variation of the chip thickness is also function of the variation of the feed speed:

$$\Delta h = (\frac{\Delta f}{2}\tan p)d\theta \tag{9}$$

Therefore, the instantaneous variation of the feed speed is the shape:

$$\Delta f = 2[(x(t) - x(t - \tau/2))\cos\theta + (y(t) - y(t - \tau/2))\sin\theta]/\tan p \tag{10}$$

and so, $\Delta C(t) = (IC_l + IC_b).2[(x(t) - x(t - \tau/2))\cos\theta + (y(t) - y(t - \tau/2))\sin\theta]/\tan p$ (11)

The motion equations become:

$$\begin{aligned} M\ddot{x}(t) + C\dot{x}(t) + Kx(t) &= \sin\theta.(IC_l + IC_b).2[(x(t) - x(t - \tau/2))\cos\theta + (y(t) - y(t - \tau/2))\sin\theta]/d\tan p \\ M\ddot{y}(t) + C\dot{y}(t) + Ky(t) &= \cos\theta.(IC_l + IC_b).2[(x(t) - x(t - \tau/2))\cos\theta + (y(t) - y(t - \tau/2))\sin\theta]/d\tan p \end{aligned} \tag{12}$$

With: $\theta = \Omega.t$ and "τ" represents the corresponding time of one revolution of the tool. Matrix writing of motion equations for the two degrees of freedom model:

$$[M]\{\ddot{U}(t)\} + [C]\{\dot{U}(t)\} + [K]\{U(t)\} = \{F(U,t)\} = [Kf]\{U(t) - U(t - \tau/2)\} \tag{13}$$

$[M]$ is the mass matrix of the tool, $[C]$ is the damping matrix of the tool, $[K]$ is the rigidity matrix of the tool, $\{U(t)\} = \begin{Bmatrix} x(t) \\ y(t) \end{Bmatrix}$ is the displacement vector and $\{F(U,t)\}$ is the cutting forces vector.

$[Kf]$ is a coupling matrix.

$$[Kf] = \frac{2(IC_l + IC_C)}{d\tan p}\begin{bmatrix} \sin\theta\cos\theta & \sin^2\theta \\ \cos^2\theta & \sin\theta\cos\theta \end{bmatrix} \tag{14}$$

By projection on the basis of modes, the equation (9) becomes:

$$\{\ddot{U}(t)\} + 2[\xi]\omega_n\{\dot{U}(t)\} + \omega_n^2\{U(t)\} = [M]^{-1}\{F(U,t)\} = [M]^{-1}[Kf]\{U(t) - U(t - \tau/2)\} \tag{15}$$

$\omega_n = \sqrt{\dfrac{K}{M}}$ is the tool natural frequency and $\xi = \dfrac{C}{2M\omega_n}$ is the modal damping coefficient.

After defining the initials data, the drill motion governing Eq. (1) can be solved by using an appropriate integration method. In the current work, we are going to solve this nonlinear delay-differential equation by using numerical methods of Newmark and Newton-Raphson.

The Newmark algorithm is an implicit method witch make possible to construct the solution in $t+\Delta t$ instant from the known vectors $\{U(t)\}, \{\dot{U}(t)\}, \{\ddot{U}(t)\}$: step-by-step time integration was used to obtain the drill tip's displacements caused by the cutting forces.

The equation of movement (1) becomes on the shape:

$$\bar{K}.U(t+\Delta t) = R(t+\Delta t) \tag{16}$$

In our case, the nonlinearity is in the second term. For solving this problem, we are going to develop a Newton-Raphson program. The characteristic of this method is that it is faster than others. Initial data are:

$$\{U(0)\} = \begin{Bmatrix} 0.000008 \\ 0.000005 \end{Bmatrix}; \{\dot{U}(0)\} = \begin{Bmatrix} 0.00002 \\ 0.00001 \end{Bmatrix}; \{\ddot{U}(0)\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}; \omega_n = 2500 rad/s; N = 900 rpm;$$

$$\xi = 1.4\%$$

The final algorithm of resolution is shown in Fig. 2.

3. NUMERICAL RESULTS

3.1 Simulation of the thrust force and the torque

The theoretical cutting model, used in the previous dynamical study, was making possible to do a parametrical study in a static way. In fact, the predicting models of the thrust and the torque have recourse to many geometrical parameters which characterize the drill [12-14].

In 1998, Mustapha Elhachimi [1] has presented this theoretical model to predict thrust and torque in drilling. The method used for the calculation of thrust and torque on the cutting lip of the drill consists of determining the element of the thrust *dFl*, and the element of the torque *dCl* for an element *dl* of the lip at an arbitrary point *M* on the edge, situated at a radius *r* from the drill axis. Force distributions along the lip are obtained using geometrical parameters, cutting conditions and properties of the machined material. Because of the symmetry around the drill axis the study will be done for one lip. The cutting geometry at a point *M* is such that the cutting speed and the tangent to the cutting edge at this point are not perpendicular. Consequently, an analysis using oblique cutting [5-6] is a better approach to describe the cutting process on this part of the drill. The geometry for a conventional twist drill will be used to determine, according to the position of the cutting point *M*, trigonometrically relationships between the different angles. The drilling investigations [15-18] have used existent models of orthogonal cutting and oblique cutting. Those models were based on the same technique of dividing the edges of the drill into elementary cutting edges, and applying there after on those edges, the oblique cutting model and the orthogonal cutting model.

The total thrust force and torque are obtained by integrating the *dCl* and *dFl* expressions, from the boundary between the lip and the chisel edge to the periphery of the drill.

$$F_l = 2\int_{d'/2}^{d/2} \frac{dF_l}{dr} dr \quad (17), \qquad C_l = 2\int_{d'/2}^{d/2} \frac{dC_l}{dr} dr \quad (18)$$

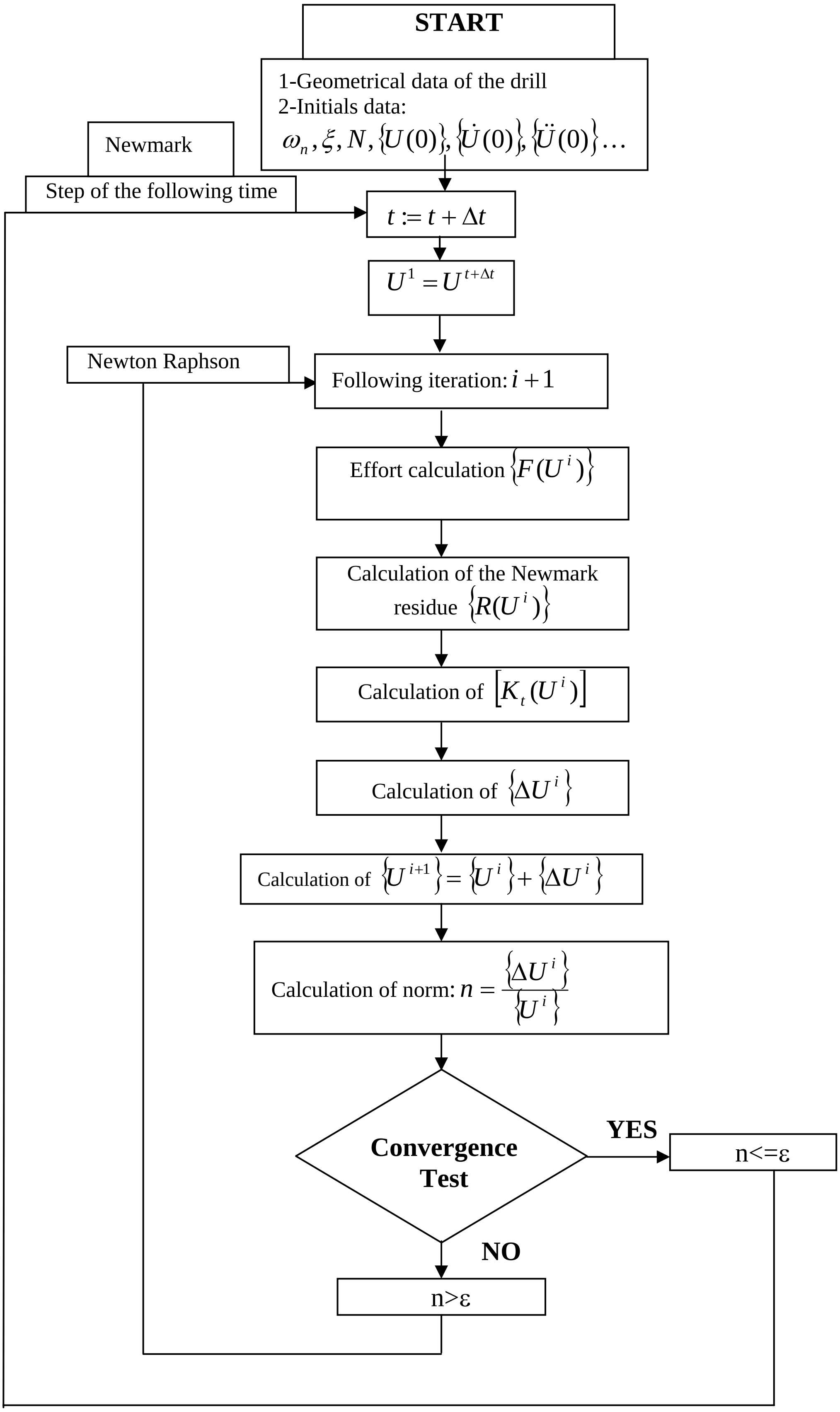


Figure 2: Algorithm of resolution.

This cutting forces model is presented by the following formulas, respectively on the cutting lips and the chisel edge [1]:

$$F_l = 2\int_{d'/2}^{d/2} k_{AB}\,\frac{f\sin p\cos\xi}{2\sin\Phi_n\cos\Phi_n}(\sin(\lambda_n-\gamma_n-\xi)\sin p-\cos p)\frac{r}{(r^2-t^2)^{1/2}}dr \tag{19}$$

$$C_l = 2\int_{d'/2}^{d/2} k_{AB}\,\frac{f\sin p\cos\xi}{2\sin\Phi_n\cos\Phi_n}\cos(\Phi_n-\gamma_n-i)\frac{r^2}{(r^2-t^2)^{1/2}}dr \tag{20}$$

$$F_C = 2\int_{r_0}^{\frac{d'}{2}} \frac{\cos(\phi_d-\gamma_d)}{\cos(\phi_d+\lambda_d-\gamma_d)}\,\frac{fk_{AB}\cos\gamma_f}{2\sin\phi_d}(\cos\gamma_f-\tan(\phi_d-\gamma_d)\sin\gamma_f)dr \tag{21}$$

$$C_C = 2\int_{r_0}^{\frac{d'}{2}} \frac{\cos(\phi_d-\gamma_d)}{\cos(\phi_d+\lambda_d-\gamma_d)}\,\frac{fk_{AB}\cos\gamma_f}{2\sin\phi_d}(\sin\gamma_f-\tan(\phi_d+\gamma_d)\cos\gamma_f)dr \tag{22}$$

The total thrust force *F* and torque *C* for the drill are obtained by adding the values from Eqs. (19) and (21) for the thrust force and Eqs. (20) and (22) for the torque:

$$F = F_l + F_C\;;\; C = C_l + C_C \tag{23}$$

This program will be inserted in the global program where the algorithm was presented in Fig. 2; the main purpose is to calculate the nonlinear cutting forces fluctuations ΔF_x and ΔF_y.

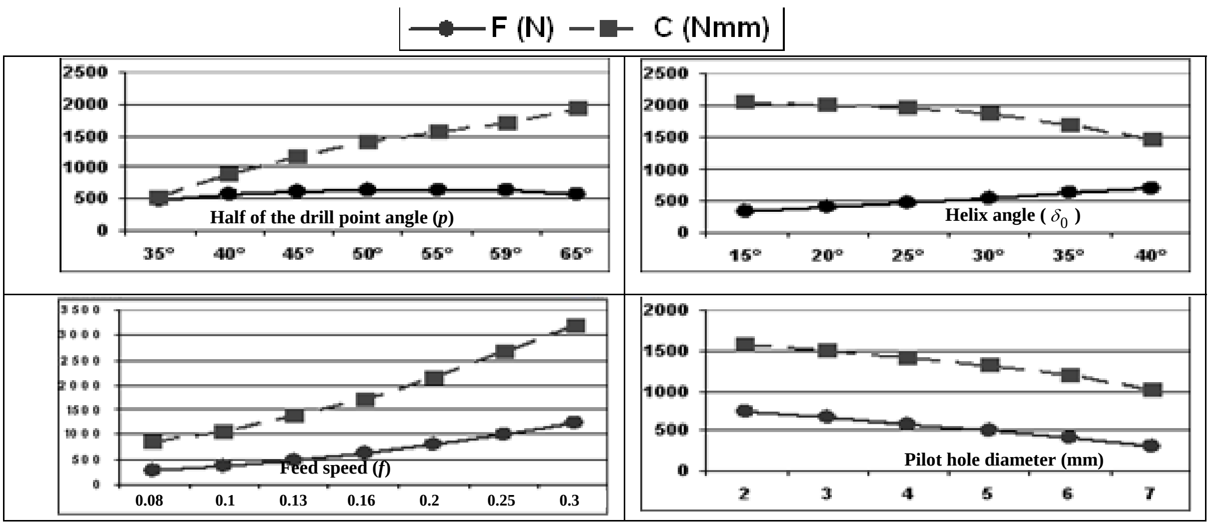


Figure 3: Total torque and thrust force on the helical drill.

Fig. 3 shows the influence of some geometrical and cutting parameters on the thrust force and the torque of the drill. The choice of these parameters can be decisive according to the adequate values of the thrust force and the torque (material to drill, power of the machine …).

3.2 Numerical results for the dynamical study

<u>Dynamical responses</u>

Dynamical study of the two degree of freedom model makes possible to analyze the vibratory behaviour of the drilling process. This study consists to solve the motion delay-differential equation of the proposed model (Eq.15) which is nonlinear and coupled. For the simulation of the different control parameters, such as the dynamical torque, feed speed variation, dynamical cutting forces, and dynamical responses in *x* and *y* directions, we consider the system with the follows dynamical characteristics: damping *ξ=1.4 %*, natural frequency $F_N=397.88Hz$.

In Fig.4 is shown the dynamical displacement in *x* and *y* directions. It permits to see the vibration evolution of the machine tool system in those two principal directions. These results was very important because it will be used in the main program to simulate the hole form derived in the next section. This figure demonstrate also that the dynamical model studied in this paper can predict the vibrations at the drill tip, this part of drill is responsible on the creation of the hole profile.

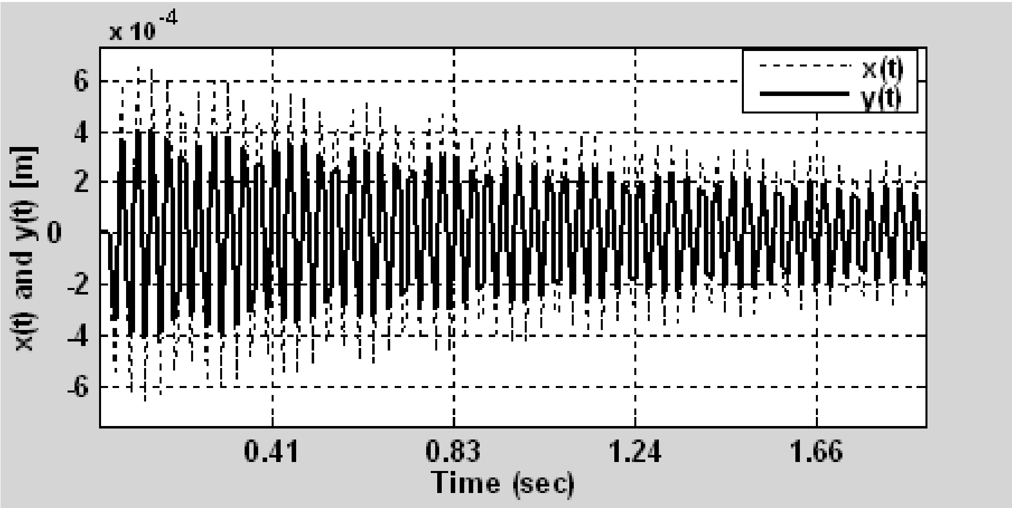


Figure 4: Dynamical displacements.

Then, these simulations permit the choice of drilling parameters according to the desired quality.

The two-degree of freedom model permits to simulate the vibration in *x* and *y* directions. This is used also to describe the relative tool position in the time. The calculation of the radial vibration in the algorithm is directly deduced about the *x(t)* and *y(t)* results. We have just to replace the values of the displacements in *x* and *y* directions at each time, in the following expression:

$$r(t) = \sqrt{x^2(t) + y^2(t)} \tag{24}$$

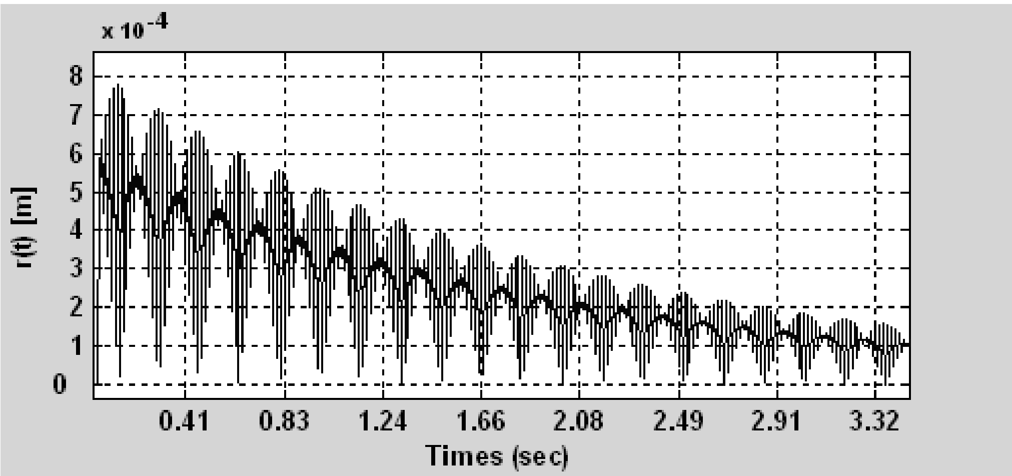


Figure 5: Dynamical radial displacement.

So, this figure shows the dynamical radial displacement. It has the same evolution in time like the displacements in *x* and *y* directions. This data characterizes the radial vibrations of the tool during the drilling operation. We can see also the beat phenomenon. This result is used to describe the global dynamical behaviour of the tool during drilling.

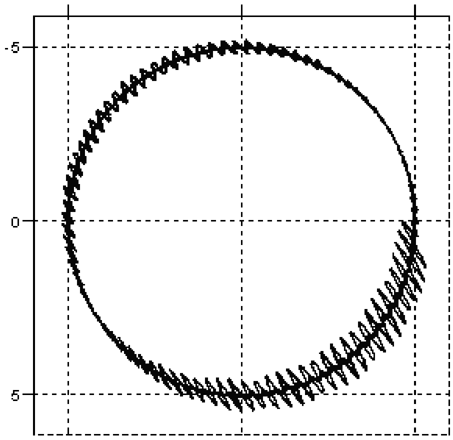
Influence of the vibrations on the drilling quality

The drilling quality is reflected by the satisfaction of some conditions like the quality of the surface regenerated and the cylindricity of the hole. Vibrations have a big influence on these conditions. The last resolution of the dynamical proposed model (Eq. 15) was make it possible to simulate the drilling result. The hole form was simulated with amplified vibrations amplitude (amplification 1000 times) in the purpose to see their evolution and their attendance on the machined surface.

The characteristics of the simulated model are following: V_C*=28 m/min*, *N=900 rpm*, *d=10 mm*, *d'=1.5 mm*, *f=0.16 mm/rev*, *p=59°*, *δ0=45°*, *depth of drilling: 8 mm.*

Material to drill: ordinary steel (k_{AB}*=360 N/mm²*).

a)



b)

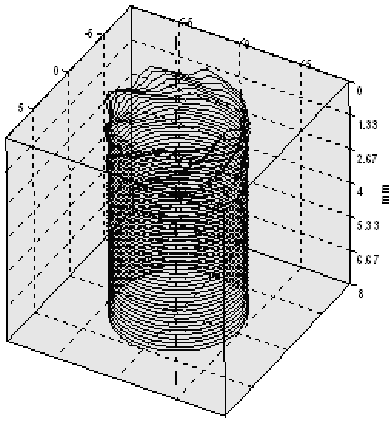


Figure 6: Influence of the vibrations on the hole quality.

Fig. 6a represents vibrations of the drilling in the first four revolutions of the tool.

Fig. 6b represents the form of the hole after drilling. In this figure, we notice that the machined surface is in the beginning of the hole more and wavier. This it explains by the fact that tool vibrations are sterner at the time of the attack of the piece. In that moment, the fluctuation of the drilling torque is very important (Fig. 7), and it affects the cutting forces.

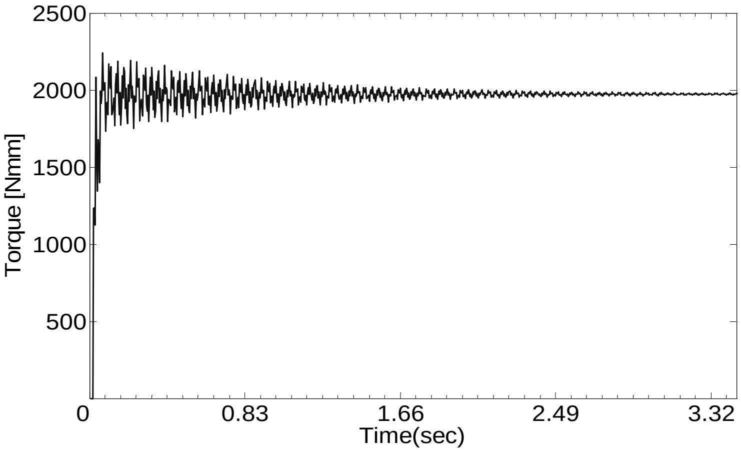


Figure 7: Drilling torque.

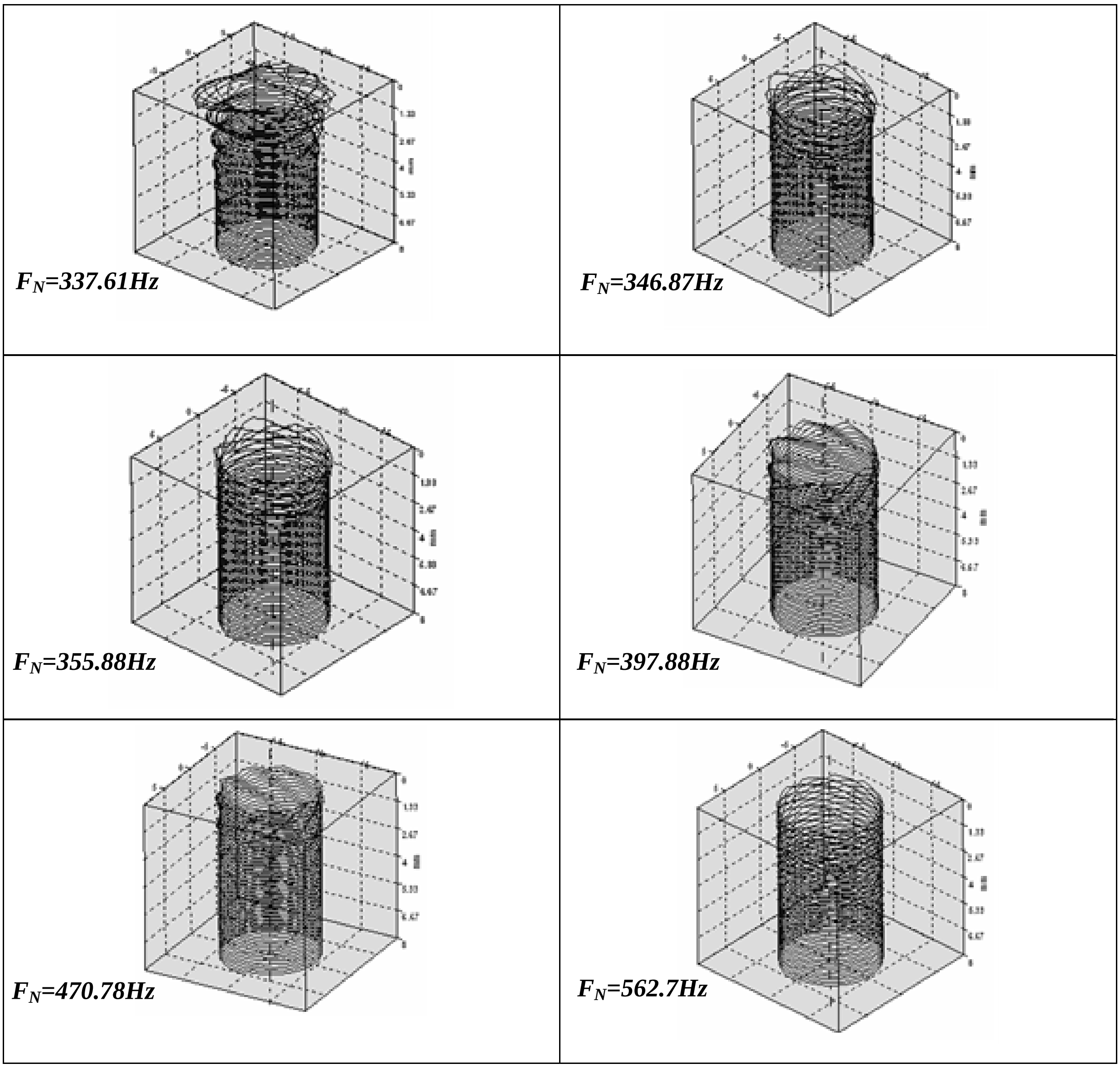


Figure 8: Effect of the natural frequency on the hole form.

In Fig. 8 is shown the Effect of the natural frequency on the simulated hole form. In fact, increasing the natural frequency reduce more and more the tool vibration because its rigidity also increases. Therefore the natural frequency might be a requirement for the choice of the hole-making machine tools.

The residual forces ΔF_x and ΔF_y generated by the cutting operation in *x* and *y* directions are the responsible to create vibrations in the considered model. In Fig. 9 is shown the evolution of the dynamical variation of those forces in *x* and *y* directions and their FFT; we can see the rotation frequency (*Fr=15 Hz*). The force model presented in this study depends on the time of *x* and *y* deflexions and the time *t*. It was used to predict the forces associated with drilling. It’s clear also in Fig. 9 on time-domain the dephasing between *x* and *y* component.

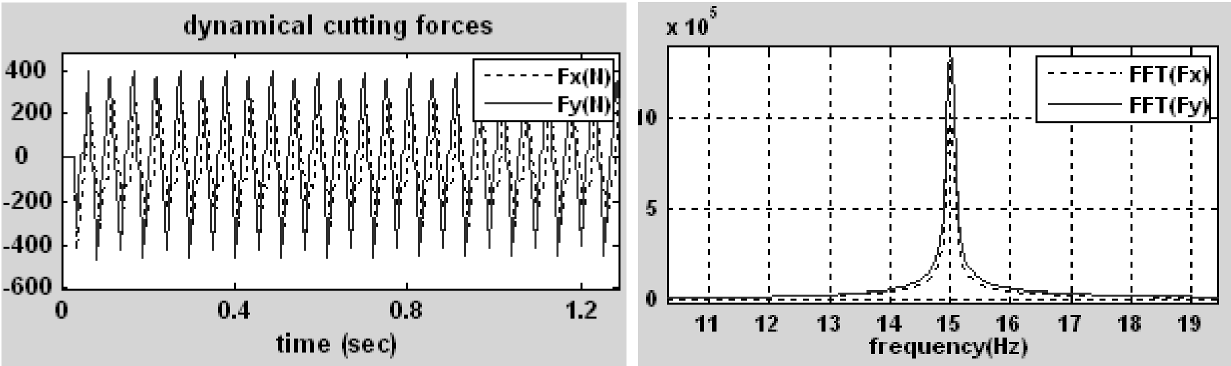


Figure 9: Dynamical cutting forces in *x* and *y* directions.

4. CONCLUSION

In this study, we elaborate the dynamical formulation of the two freedom degrees system in drilling. This model is based on a number of assumptions such as: the work piece is rigid, the tool is rigid in the *z* direction, the effect of the chip is ignored and the body of the tool will not touch the hole. The vibration phenomenon in this model represents the result of the variation of the residual forces ΔF_x *and* ΔF_y generated by the cutting operation in *x* and *y* directions. Each of them depends at a time of the instantaneous displacement *x* and *y* and the time. The *x* and *y* displacements was directly affected by the natural frequency. In this model, the problem is nonlinear and coupled between *x* and *y* directions. The numerical resolution of this dynamical model represents the main theoretical way to discover the dynamical behaviour of drilling process. It permits to view the evolution of the dynamical displacements in two principal directions *x* and *y* (dynamical responses). It permits also to predict the dynamical radial displacement and the hole form with amplified vibrations. Also it permits to show the influence of the natural frequency on the tool vibrations and the hole form.

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