

# FINITE ELEMENT SOLUTION OF DYNAMIC RESPONSE OF HELICAL SPRINGS

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## Abstract

A numerical solution is presented to describe wave propagations in axially impacted helical springs. The governing equations for such problem are two coupled hyperbolic, partial differential equations of second order. The axial and rotational strains and velocities are considered as principal dependent variables. Since the governing equations are non-linear, the solution of the system of equations can be obtained only by some approximate numerical simulation. The finite element method, based on the Galerkin formulation, is applied to discretize the mathematical equations leading to a non-linear system of equations solved by an iterative Gauss substitution method. The computed results describe the evolution of axial and rotational strains and velocities, in different sections of the spring and show the interaction between axial and rotational waves. To validate the reliability of the model presented herein, the numerical results are compared with those of other workers obtained by the method of characteristics.

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**Key Words:** Helical Spring, Dynamic Response, Strain, Finite Element Method, Non-linear Behaviour

## 1. INTRODUCTION

The helical spring is the simplest element which is used in the mechanical engineering and which can be found in many apparatuses, as the balances, the brakes and the suspensions. It makes it possible to maintain a tension or a force in a mechanical system, to absorb the shocks and to reduce the vibrations. To simplify the analysis; it is generally assumed that the material is elastic. The design of helical springs requires two stages as the static and dynamic. The analytical solution of the static response of cylindrical helical springs subjected to large deflections was achieved by Love [1].

The dynamic response of helical springs can be broadly characterized as linear or non-linear. In the past, the dynamic response is treated by restricting the analysis to small displacements about an equilibrium position. Therefore, the oscillatory system can be assumed to be linear. Costello [7] presents a work on the significance of torsional oscillations on the radial expansion of helical springs. In this work a linear theory was presented and the analytical solution, obtained by the Laplace transform, did indicate rather large radial expansion under impact. Ayadi and Hadj-Taïeb [2, 3] resolved numerically the linear model of Costello by using the method of characteristics and the finite difference method of Lax-Wendroff.

When a helical spring is subjected to a rather large impact loading, significant torsional oscillations can occur in the spring and the behaviour of the spring becomes non-linear. The one dimensional equations of motion, governing this behaviour, are derived in an article by Phillips and Costello [12].

Stokes [14] conducted an analytical and experimental program to investigate the radial expansion of helical springs due to longitudinal impact.

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