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UNCERTAINTY, DUALISM AND INVERSE REACHABLE SETS

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Abstract
A specification in the language of the theory of categories is given for dynamic models with uncertainty. For continuous models the uncertainty is treated by means of differential inclusions. The reversibility and the inverse problem are considered. It is shown that models with uncertainty are not reversible (cannot be solved simply reversing the time). The inverse problem for such models is solved using the concept of dualism in the theory of categories. The categorical language is much more abstract and the categorical specification may work in continuous case as well in the discrete event simulation.
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Key Words: Uncertainty, Differential Inclusion, Dynamic System, Duality, Category Theory

1. INTRODUCTION

While constructing models, the uncertainty is being treated in many different ways. Roughly speaking, by *uncertainty* we understand a lack of exact information about some model parameters, or even about the model structure. Mostly, the uncertain parameters are represented in the model as random variables, which add the stochastic properties to the model. As there is a huge research on stochastic systems already done, this approach works well, provided the specification of the randomness is known. In general, to use a random variable we must know something about the basic parameters of the corresponding probability distribution. Unfortunately, this is not always the case. The mean value is frequently a confusing and little informative, and the standard deviation does not describe the random variable sufficiently enough. The density function is rarely known, and it may be inexistent at all. This is, for example, the case when the uncertain parameter represents false information intentionally inserted into the modelled system (as in stock market models).

There is no room here to mention a huge number of publications on uncertainty problems. See Bargiela and Hainsworth [1] or Bargiela [2] for an example of a quite different approach to the uncertainty management in water distribution systems, for example.

If we construct a continuous ODE (Ordinary Differential Equations) model, then the general description of its dynamics is, in most cases, given by the equation:

$$\frac{dx}{dt} = f(x(t),t) \ , \quad x(0) = x_0 \tag{1}$$

where $x(t)$ is the system state (a scalar, a point of the R^n or a more abstract state space), and x_0 is the initial condition. Now, suppose that the value of f is charged with some uncertainty. Our, non-stochastic approach to the uncertainty problem is to replace (1) with a differential inclusion:

$$\frac{dx}{dt} \in F(x(t),t) \ , \quad x(0) \in X_0 \tag{2}$$

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