

# EVALUATION OF COMBINED PARETO MULTIOBJECTIVE DIFFERENTIAL EVOLUTION ON TUNEABLE PROBLEMS

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## Abstract

Many optimization problems in engineering involve the satisfaction of multiple objectives within the limits of certain constraints. Methods of evolutionary multi-objective algorithms (EMOAs) have been proposed and applied to solve such problems. Recently, a combined Pareto multi-objective differential evolution (CPMDE) algorithm was proposed. The algorithm combines Pareto selection procedures for multi-objective differential evolution to implement a novel selection scheme. The ability of CPMDE in solving unconstrained, constrained and real optimization problems was demonstrated and competitive results obtained from the application of CPMDE suggest that it is a good alternative for solving multi-objective optimization problems. In this work, CPMDE is further tested using tuneable multi-objective test problems and applied to solve a real world engineering design problem. Results obtained herein further corroborate the efficacy of CPMDE in multi-objective optimization.

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**Key Words:** Multi-Objective Optimization, Constraints, Differential Evolution, Tuneable Test Beds, Evolutionary Algorithms

## 1. INTRODUCTION

In most practical decision-making problems, the presence of multiple objectives or multiple criteria is evident [1]. Due to the multi-criteria nature of most real-world problems, multi-objective optimization problems (MOOPs) are very common particularly in engineering and scientific designs and applications. MOOPs involve multiple often conflicting objectives, which are to be optimized simultaneously. There is no single optimal solution to this class of problems; rather, the solution consists of a group of alternative trade-off solutions called Pareto-optimal or non-inferior solutions which must be considered equivalent in the absence of specialized information concerning the relative importance of the objectives [2, 3].

Evolutionary algorithms (EAs) are population-based meta-heuristic optimization algorithms that use biology-inspired mechanisms like mutation, crossover, natural selection and survival of the fittest in order to refine a set of solution candidates iteratively. EAs have often performed well approximating solutions to all types of problems because they ideally do not make any assumption about the underlying fitness landscape [4, 5]. Since EAs deal with a group of candidate solutions, it seems natural to use them in MOOPs to find a group of optimal solutions. Indeed, the applications of evolutionary multi-objective algorithms (EMOAs) to the solution of real world MOOPs have been demonstrated and they have been found very efficient in solving these classes of problems [6-8]. The development of the theory of EMOA in recent years has spurred researches in the field of development of EMOAs for the solution of real-world problems. Over the past decades, several studies involving the extension of evolutionary algorithms to solve multi-objective numerical optimization problems have been reported [9-11].

Differential evolution (DE) is currently one of the most popular heuristics being used for solving single-objective optimization problems in continuous search spaces. Due to its reported successes on a myriad of problems, its use has been extended to other types of problem domains, including multi-objective optimization [6, 12]. In recent times, several

researches extending the application of DE for finding solutions in the multi-objective problem domains have been reported in the literatures [13-16]. For instance Robic and Filipic [17] proposed three strategies of a Pareto-based differential evolution for multi-objective optimization (DEMO). The suggested strategies are DEMO/parent, DEMO/closest/dec and DEMO/closest/obj. In DEMO/parent, a child replaces the parent at the same index if it dominates that parent while in DEMO/closest/dec, the child replaces the closest parent to it in the decision space. DEMO/closest/obj replaces a parent with a child in the objective function space. The performances of the strategies of DEMO were compared with six other methods of EMOAs on five benchmark test beds. It was found that DEMO outperformed the other algorithms especially those based on other EAs such as genetic algorithm (GA) and evolution strategies (ES). It was thus concluded that DEMO may be adopted as an alternative for solving MOOPs.

In [18], a combined Pareto multi-objective differential evolution (CPMDE) algorithm is introduced. The algorithm combines Pareto selection procedures for multi-objective differential evolution to implement a novel selection scheme at each generation. The performance of CPMDE was evaluated using common difficult test problems obtained from multi-objective evolutionary computation literatures. The ability of the algorithm in solving unconstrained, constrained and real optimization problems was demonstrated and competitive results obtained from its application suggest that it is a good alternative for solving multi-objective optimization problems. However, [1] has argued that most of these test problems are not tuneable and it is difficult to establish the feature of an algorithm that has been tested. Based on this argument, the author presented a systematic procedure of designing test problems for unconstrained and constrained multi-objective evolutionary optimization and constructed a set of six difficult test problems. These problems have further been studied by researcher in the field [9, 10, 14, 17]. Motivated by the preceding argument, we further test CPMDE using five (continuous) tuneable unconstrained multi-objective test problems and one constrained test problem. Furthermore, we apply CPMDE to solve a real world engineering design problem. Results obtained herein further corroborate the efficacy of CPMDE in solving multi-objective optimization problems.

The remainder of this paper is structured as follows. In section 2 we present a brief description of the CPMDE algorithm. A more thorough description of CPMDE can be found in [18]. Section 3 present methodologies adopted in benchmarking CPMDE using tuneable unconstrained multi-objective test beds, a constrained test problem and an application to the solution of an engineering design problem, while section 4 presents the results of the benchmark and evaluation of CPMDE. The paper is concluded in section 5.

## **2. THE CPMDE ALGORITHM**

In CPMDE, the combined population of trial and target solutions at the end of every iteration is checked for non-dominated solutions. Solutions that will proceed to the next generation are selected using a combined Pareto ranking and Pareto dominance selection scheme [6]. Diversity among solutions in the obtained non-dominated set is promoted using a harmonic average crowding distance measure [19]. Furthermore, boundary constraints are handled using the bounce-back strategy [12] while equality and inequality constraints are handled using the constrained-domination technique suggested by [1]. The CPMDE algorithm is summarized as follows [18]:

1. Input the required DE parameters like number of individuals in the population  $Np$ , mutation scaling factor  $F$ , crossover probability  $Cr$ , maximum number of iterations/generations  $gMax$ , number of objective functions  $k$ , number of decision variables/parameters  $D$ , upper and lower bounds of each variable, etc.
2. Initialize all solution vectors randomly within the limits of the variable bounds.
3. Set the generation counter,  $g = 0$ .
4. Generate a trial population of size  $Np$  using DE's mutation and crossover operations [12].
5. Perform a domination check on the combined trial and target population and mark all non-dominated solutions as "non-dominated" while marking others as "dominated".
6. Play domination tournament at each population index.
  - i. If the trial solution is marked "non-dominated" and the target is marked "dominated" then the trial vector replaces the target vector.
  - ii. If the trial solution is marked "dominated" and the target is marked "non-dominated" then the trial vector is discarded.
  - iii. If both solutions are marked "dominated", then replace the target vector if it is dominated by the trial vector or if they are non-dominated with respect to each other.
  - iv. If both vectors are marked "non-dominated", then note down the index and proceed to the next index. When all solutions marked "non-dominated" from steps i – iii above are installed in the next generation, then sort out all solutions noted in step iv one at a time using the harmonic average crowding distance measure [19]. The solution with a greater harmonic average distance is selected to proceed to the next generation.
7. Increase the generation counter,  $g$ , by 1. i.e.  $g = g + 1$ .
8. If  $g < gMax$ , then go to step 4 above else go to step 9.
9. Remove the dominated solutions in the last generation.
10. Output the non-dominated solutions.

\*Note domination checks are performed using the naive and slow method [1].

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Source: [18]

### **3. EVALUATING AND BENCHMARKING CPMDE**

The performance of CPMDE is compared with 13 state-of-the-art EMOAs on five unconstrained benchmark test beds and one constrained test problem. Furthermore, the performance of CPMDE on an engineering two-bar truss design problem is demonstrated. Other algorithms used in benchmarking CPMDE in this study include NSGA-II (real coded), NSGA-II (binary coded), SPEA, PAES, PDEA, MODE, MODE-E (MODE with external archive and crowding distance measure), MOPSO, SDE, DEMO/parent, DEMO/closest/dec, DEMO/closest/obj and MDEA.

#### **3.1 Description of benchmark test problems**

Five Zitzler-Deb-Thiele (ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6) test problems, which are common tuneable difficult benchmark problems used in the literatures [9, 17, 20, 21] are

chosen for evaluating the performance of CPMDE on unconstrained optimization problems. These are bi-objective problems in which both objectives are to be minimized. Each problem poses a different type of difficulty to EMOAs. The definitions and descriptions of all test functions are taken from literatures [1, 8, 14, 17] and summarized in Table I.

Table I: Summary of benchmark test problems.

Problem	$n$	Variable bounds	Objective functions and constraints	Pareto-optimal solutions	Comments
ZDT1	30	[0, 1]	$f_1(x) = x_1$ $f_2(x) = g(x)\left(1 - \sqrt{f_1(x)/g(x)}\right)$ $g(x) = 1 + 9\left(\sum_{i=2}^n x_i\right)/(n-1)$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, 3, \dots, n$	Convex
ZDT2	30	[0, 1]	$f_1(x) = x_1$ $f_2(x) = g(x)\left(1 - (f_1(x)/g(x))^2\right)$ $g(x) = 1 + 9\left(\sum_{i=2}^n x_i\right)/(n-1)$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, 3, \dots, n$	Non-convex
ZDT3	30	[0, 1]	$f_1(x) = x_1$ $f_2(x) = g(x)\left[1 - \left[\frac{x_1}{g(x)} - \frac{x_1}{g(x)}\sin(10\pi x_1)\right]\right]$ $g(x) = 1 + 9\left(\sum_{i=2}^n x_i\right)/(n-1)$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, 3, \dots, n$	Convex, Discontinuous  Not all points in $0 \leq x_i \leq 1$ lie on the Pareto-optimal front
ZDT4	10	$x_1 \in [0,1]$ $x_i \in [-5,5]$ $i = 2, 3, \dots, n$	$f_1(x) = x_1$ $f_2(x) = g(x)\left[1 - \sqrt{\frac{x_1}{g(x)}}\right]$ $g(x) = 1 + 10(n-1) + \sum_{i=2}^n (x_i^2 - 10\cos(4\pi x_i))$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, 3, \dots, n$	Convex,  Deceptive, Multiple local, Optimal fronts
ZDT6	10	[0, 1]	$f_1(x) = 1 - e^{-4x_1} \sin^6(6\pi x_1)$ $f_2(x) = g(x)\left[1 - \left(\frac{f_1(x)}{g(x)}\right)^2\right]$ $g(x) = 1 + \frac{9}{n-1} \sum_{i=2}^n x_i$	$x_1 \in [0,1]$ $x_i = 0$ $i = 2, 3, \dots, n$	Nonconvex, Nonuniformly spaced solutions on Pareto front, Low density of solutions near Pareto front
CONSTR	10	$x_1 \in [0.1,1]$ $x_2 \in [0,5]$	$f_1(x) = x_1$ $f_2(x) = \frac{1+x_2}{x_1}$ <i>Subject to:</i> $g_1(x) = x_2 + 9x_1 \geq 6$ $g_2(x) = -x_2 + 9x_1 \geq 1$	$0.39 \leq x_1 \leq 0.67$ $\therefore x_2 = 6 - 9x_1$ and $0.67 \leq x_1 \leq 1.00$ $\therefore x_2 = 0$	Convex, Constrained, Segmented Pareto front

ZDT1 is a 30-variable problem having a convex Pareto-optimal front. The difficulty an EMOA may face on this problem is in tackling the large number of decision variables. ZDT2 is also a 30-variable problem but has a non-convex Pareto-optimal front. The non-convexity of the front is the major difficulty posed here. ZDT3 is a 30-variable problem having a number of disconnected Pareto-optimal fronts. Finding uniform spread of solutions on all discontinuous regions is the challenge in this problem. ZDT4 is a 10-variable problem with  $21^9$  local optimal fronts. Escaping all local non-dominated fronts to converge to the global optimal front is a real challenge in this problem. ZDT6 is a 10-variable problem having a non-

convex Pareto-optimal front with non-uniform distribution of solutions on the front. Finding a uniform spread of solution on this non-convex front poses a challenge to EMOAs.

To evaluate the performance of CPMDE on constrained optimization problems, the problem CONSTR is used [1, 8]. This is a bi-objective problem with two constraints. Both of the objectives are to be minimized. CONSTR has a convex Pareto-optimal front with two distinct segments. Finding uniform spread of solutions on both segments while satisfying both constraints is a challenge in this problem.

### 3.2 Performance measures

Performance measures that exist for EMOA evaluation are multifarious [3, 22]. However, in order to provide a uniform basis for comparison of EMOAs used in this study, the three performance measures reported in the published studies were adopted [9, 14, 17]. The generational distance and convergence metric are used to evaluate convergence to the global Pareto-optimal front while the diversity metric is employed to measure the spread of solutions on the obtained non-dominated front.

#### Generational distance

This is the average distance of the non-dominated set of solutions in a set  $Q$  from a set of chosen Pareto-optimal solutions. It is computed using eq. (1):

$$GD = \frac{\left( \sum_{i=1}^{|Q|} d_i^p \right)^{\frac{1}{p}}}{|Q|} \quad (1)$$

For  $p = 2$ ,  $d_i$  is the Euclidean distance (in the objective space) between the solutions of  $Q$  and the nearest member in the true Pareto-front. An algorithm having a small value of  $GD$  is better [1].

#### Convergence metric

This is a special case of the generational distance where  $p = 1$ . Convergence metric is computed using eq. (2) [17]:

$$\Upsilon = \frac{\sum_{i=1}^{|Q|} d_i}{|Q|} \quad (2)$$

#### Diversity metric

This metric measures the extent and spread of solutions in the obtained non-dominated front. It is computed using eq. (3):

$$\Delta = \frac{\sum_{m=1}^M d_m^e + \sum_{i=1}^{|Q|-1} |d_i - \bar{d}|}{\sum_{m=1}^M d_m^e + (|Q| - 1)\bar{d}} \quad (3)$$

where  $d_i$  is the Euclidean distance (in the objective space) between consecutive solutions in the obtained non-dominated front  $Q$ , and  $\bar{d}$  is the average of these distances. The parameter  $d_m^e$  is the Euclidean distances between the extreme solutions of the Pareto front and the boundary solution of the obtained non-dominated front  $Q$  with respect to each objective  $m$ . An algorithm with a smaller value of diversity metric  $\Delta$  is better [1].

### 3.3 Experimental setup

In this study, DE/rand/1/bin variant of DE was used as the base for CPMDE.  $Cr$  and  $F$  were set at 0.3. Population size  $Np$  was set to 100 and the algorithm was run for a maximum number of generations,  $gMax = 250$ . A set of 500 uniformly spaced solutions were taken from the Pareto-optimal set for computation of all metrics. Averages and variances of metric values over 10 runs are reported in this study. For comparison of the performance of CPMDE with NSGA-II on the problem CONSTR, the following parameters are used:  $Cr = F = 0.3$ ,  $Np = 40$  and  $gMax = 200$ . Harmonic average crowding distances are computed using the two nearest neighbours on either sides of a solution.

### 3.4 Two-bar truss design problem

To demonstrate the applicability of CPMDE in solving real-world optimization problems, the algorithm was applied to design a two-bar truss system. A problem originally studied by [23] using the  $\epsilon$ -constraint method and further studied by [24] using NSGA-II is adopted here. A schematic representation of the two-bar truss is depicted in Fig. 1.

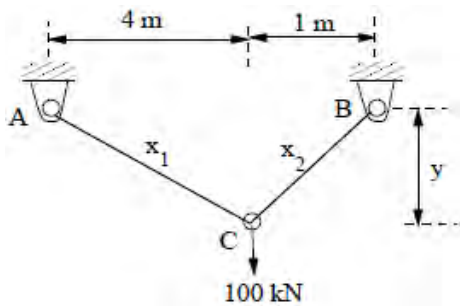


Figure 1: A schematic diagram of a two-bar truss [24].

The truss is designed to carry a certain load without elastic failure. Thus, in addition to the objective of designing the truss for minimum volume (which is equivalent to designing for minimum cost of fabrication); there are additional objectives of minimizing stresses in each of the two members AC and BC. The two-objective constrained optimization problem for three decision variables  $y$  (vertical distance between B and C in m),  $x_1$  (cross-sectional area of AC in  $m^2$ ) and  $x_2$  (cross-sectional area of BC in  $m^2$ ) is formulated as follows [24]:

$$\text{Objective function 1 (Minimize volume): } f_1(x_1, x_2, y) = x_1\sqrt{16 + y^2} + x_2\sqrt{1 + y^2}$$

$$\text{Objective function 2 (Minimize stress): } f_2(x_1, x_2, y) = \max(\sigma_{AC}, \sigma_{BC})$$

$$\text{where: } \sigma_{AC} = \frac{20\sqrt{16 + y^2}}{yx_1}, \quad \sigma_{BC} = \frac{80\sqrt{1 + y^2}}{yx_2}$$

$$\text{Subject to: } \max(\sigma_{AC}, \sigma_{BC}) \leq 1(10^5)$$

$$\text{Bound constraints: } 1 \leq y \leq 3, \quad 0 \leq x_1, x_2 \leq 0.01$$

$\sigma_{AC}$  and  $\sigma_{BC}$  are the stresses in members AC and BC respectively. On this problem, the following settings are used for CPMDE:  $Cr = 0.9$ ,  $F = 0.5$ ,  $Np = 100$  and  $gMax = 100$ .

## 4. RESULTS AND DISCUSSION

The mean and variance of the convergence metric on the unconstrained test beds, over 10 runs of CPMDE are reported in Table II while those of the diversity metric are presented in Table

III. The convergence metric for PDEA was not available; therefore for comparative study on this algorithm, the available generational distance metric is extracted from literatures and presented along with those of DEMO and CPMDE in Table IV. Diversity metrics for MODE on all test functions are not available because they were not calculated in the original study. Also, the performance metric for MOPSO on the test problem ZDT4 is not available. The authors reported that this algorithm failed on this test bed.

Table II: Convergence metrics on unconstrained test beds.

Algorithm	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6
NSGA-II (real coded)	0.033482±0.004750	0.072391±0.031689	0.114500±0.004940	0.513053±0.118460	0.296564±0.013135
NSGA-II (binary coded)	0.000894±0.000000	0.000824±0.000000	0.043411±0.000042	3.227636±7.307630	7.806798±0.001667
SPEA	0.001799±0.000001	0.001339±0.000000	0.047517±0.000047	7.340299±6.572516	0.221138±0.000449
PAES	0.082085±0.008679	0.126276±0.036877	0.023872±0.000010	0.854816±0.527238	0.085469±0.006664
PDEA	N/A	N/A	N/A	N/A	N/A
MODE	0.005800±0.000000	0.005500±0.000000	0.021560±0.000000	0.638950±0.500200	0.026230±0.000861
MODE-E	0.001999±0.000000	0.001554±0.000000	0.002642±0.000000	0.030689±0.004867	0.005998±0.000005
MOPSO	0.019659±0.000012	0.017093±0.000133	0.030469±0.000067	N/A	0.751692±0.151000
SDE	0.002741±0.000385	0.002203±0.000297	0.002741±0.000120	0.100100±0.446200	0.000624±0.000060
DEMO/parent	0.001083±0.000113	0.000755±0.000045	0.001178±0.000059	0.001037±0.000134	0.000629±0.000044
DEMO /closest/dec	0.001113±0.000134	0.000820±0.000042	0.001197±0.000091	0.001016±0.000091	0.000630±0.000021
DEMO /closest/obj	0.001132±0.000136	0.000780±0.000035	0.001236±0.000091	0.041012±0.063920	0.000642±0.000029
MDEA	0.000921±0.000005	<b>0.000640</b> ±0.000000	0.001139±0.000024	0.048962±0.536358	<b>0.000436</b> ±0.000055
CPMDE	<b>0.000755</b> ±0.000000	0.000775±0.000000	<b>0.000916</b> ±0.000000	<b>0.000731</b> ±0.000000	0.000584±0.000000

Table III: Diversity metrics on unconstrained test beds.

Algorithm	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6
NSGA-II (real coded)	0.390307±0.001876	0.430776±0.004721	0.738540±0.019706	0.702612±0.064648	0.668025±0.009923
NSGA-II (binary coded)	0.463292±0.041622	0.435112±0.024607	0.575606±0.005078	0.479475±0.009841	0.644477±0.035042
SPEA	0.784525±0.004440	0.755184±0.004521	0.672938±0.003587	0.798463±0.014616	0.849389±0.002713
PAES	1.229794±0.000742	1.165942±0.007682	0.789920±0.001653	0.870458±0.101399	1.153052±0.003916
PDEA	0.298567±0.000742	0.317958±0.001389	0.623812±0.000225	0.840852±0.035741	0.473074±0.021721
MODE	N/A	N/A	N/A	N/A	N/A
MODE-E	0.306235±0.001130	0.298449±0.000580	0.504275±0.000200	0.338330±0.003676	0.335594±0.019000
MOPSO	0.586728±0.001480	0.689580±0.038200	0.594200±0.001150	N/A	0.935686±0.018500
SDE	0.382890±0.001435	0.345780±0.003900	0.525770±0.043030	0.436300±0.110000	0.361100±0.036100
DEMO/parent	0.325237±0.030249	0.329151±0.032408	0.309436±0.018603	0.359905±0.037672	0.442308±0.039255
DEMO /closest/dec	0.319230±0.031350	0.335178±0.016985	0.324934±0.029648	0.359600±0.026977	0.461174±0.035289
DEMO /closest/obj	0.306770±0.025465	0.326821±0.021083	0.328873±0.019142	0.407225±0.094851	0.458641±0.031362
MDEA	0.283708±0.002938	0.450482±0.004211	<b>0.299354</b> ±0.023309	0.406382±0.062308	0.305245±0.019407
CPMDE	<b>0.241173</b> ±0.000077	<b>0.266395</b> ±0.000379	0.353734±0.000522	<b>0.203378</b> ±0.000400	<b>0.217533</b> ±0.000234

Table IV: Generational distance metrics on unconstrained test beds.

Algorithm	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6
PDEA	0.000615±0.000000	0.000652±0.000000	0.000563±0.000000	0.618258±0.826881	0.023886±0.003294
DEMO/parent	0.000230±0.000048	0.000091±0.000004	0.000156±0.000007	0.000202±0.000053	0.000074±0.000004
DEMO/closest/dec	0.000242±0.000028	0.000097±0.000004	0.000162±0.000013	0.000179±0.000048	0.000075±0.000002
DEMO/closest/obj	0.000243±0.000050	0.000092±0.000004	0.000169±0.000017	0.004262±0.006545	0.000076±0.000003
CPMDE	<b>0.000086±0.000000</b>	<b>0.000087±0.000000</b>	<b>0.000107±0.000000</b>	<b>0.000085±0.000000</b>	<b>0.000068±0.000000</b>

Reported values of generational distances, convergence and diversity metrics for other algorithms used in benchmarking CPMDE are taken from correlative literatures [9, 14, 17, 20, 21] and presented in the respective tables. Best mean results are shown in boldface. Fig. 2 depicts the convergence of the obtained non-dominated front to the true Pareto-optimal front in problems ZDT1, ZDT2, ZDT3, ZDT4, ZDT6 and CONSTR respectively. The values of the test metrics are indicated on the respective plots. Fig. 3 shows the performance of NSGA-II and CPMDE, respectively, for 200 generations on the CONSTR problem. Fig. 4 shows the results obtained by NSGA-II and CPMDE on the two-bar truss design problem.

From the results in Tables II – IV, it is found that CPMDE outperformed all other algorithms on ZDT1 test bed as it produced the minimum values of all test metrics in all cases ( $Y = 0.000755$ ,  $GD = 0.000086$ ,  $\Delta = 0.241173$ ). Binary coded NSGA-II performed second with respect to convergence ( $Y = 0.000894$ ) while MDEA was the second best with respect to diversity ( $\Delta = 0.283708$ ). MDEA and DEMO/parent showed slightly better convergence property on ZDT2, however, CPMDE performed best with respect to diversity preservation this test best reporting a value of  $\Delta = 0.266395$ . Therefore the performance of CPMDE on this problem is comparable to MDEA and DEMO but better than those of other algorithms. Table II shows that CPMDE performed best in converging to the Pareto-optimal front of ZDT3 with a convergence metric,  $Y = 0.000916$ . However, the reported diversity metric for MDEA and all versions of DEMO were better. Hence, the performance of CPMDE is comparable with MDEA and DEMO on this test bed. The advantage of CPMDE in converging to the global Pareto-optimal front in deceptive multi-modal functions is amply demonstrated on test problem ZDT4. Here, CPMDE outperformed all other algorithms in convergence and diversity ( $Y = 0.000731$ ,  $GD = 0.000085$ ,  $\Delta = 0.203378$ ). The runner-ups in this case are DEMO/closest/dec ( $Y = 0.001016$ ) on convergence and MODE-E on diversity ( $\Delta = 0.338330$ ). The convergence metric on this problem is clearly smaller than those of other algorithms. On ZDT6, CPMDE performed second best with a convergence metric ( $Y = 0.000584$ ) which is slightly smaller than 0.000584 reported for MDEA. However, CPMDE produced the best diversity ( $\Delta = 0.217533$ ) on this problem. On all unconstrained problems, CPMDE produces variance values of zero for convergence metrics and generational distances (Tables II and IV). This suggests that CPMDE is reliable and stable in converging to the true Pareto front on these test beds.

By inspection, Fig. 3 shows that while NSGA-II was able to produce solutions covering roughly 80 % of the Pareto-optimal front for 200 iterations on the CONSTR test problem, CPMDE spanned the entire front with better convergence property. Therefore, CPMDE outperforms NSGA-II which is one of the state-of-the-art algorithms on this problem.

On the two-bar truss design problem,  $\epsilon$ -constraint found only five solutions with spread (0.004445 m<sup>3</sup>, 89983 kPa) - (0.004833 m<sup>3</sup>, 83268 kPa) while NSGA-II found many solutions in the range (0.00407 m<sup>3</sup>, 99755 kPa) - (0.05304 m<sup>3</sup>, 8439 kPa). CPMDE was also able to find many solutions spanning the range (0.00408 m<sup>3</sup>, 98787 kPa) - (0.07384 m<sup>3</sup>, 8433 kPa). If



minimization of stress is important, NSGA-II finds a solution with stress as low as 8439 kPa, whereas the  $\epsilon$ -constraint method found a solution with minimum stress of 83268 kPa, an order of magnitude higher than that found in NSGA-II [9]. CPMDE found a solution with minimum stress of 8433 kPa which is slightly less than that found by NSGA-II, thus, the performance of CPMDE is comparable with NSGA-II on this problem. CPMDE produces many quality non-dominated solutions on the Pareto-optimal front of this problem in a single run (Fig. 4). This shows that CPMDE can perform well on real-world engineering problems.

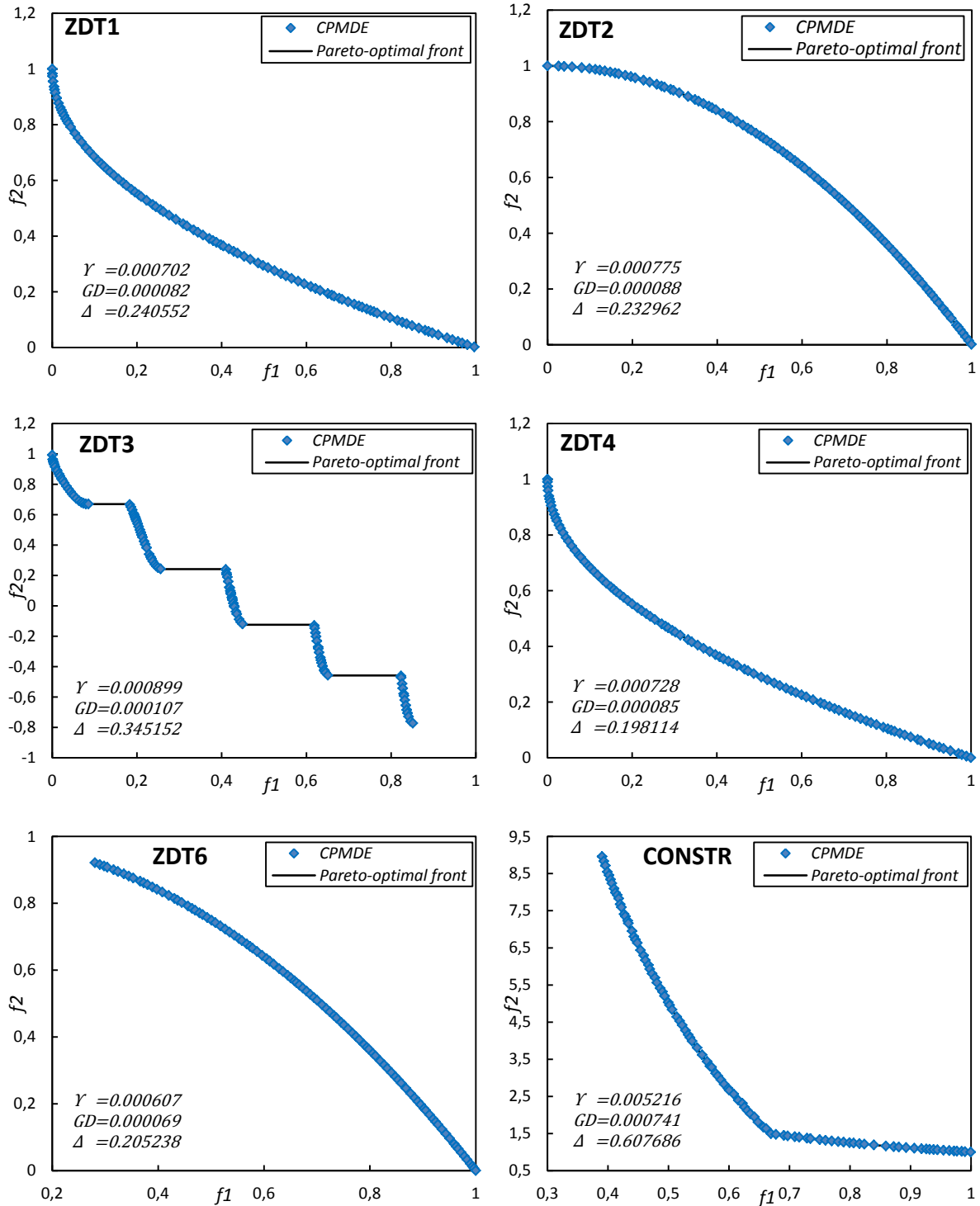


Figure 2: Convergence of CPMDE non-dominated front to the true Pareto-optimal front.

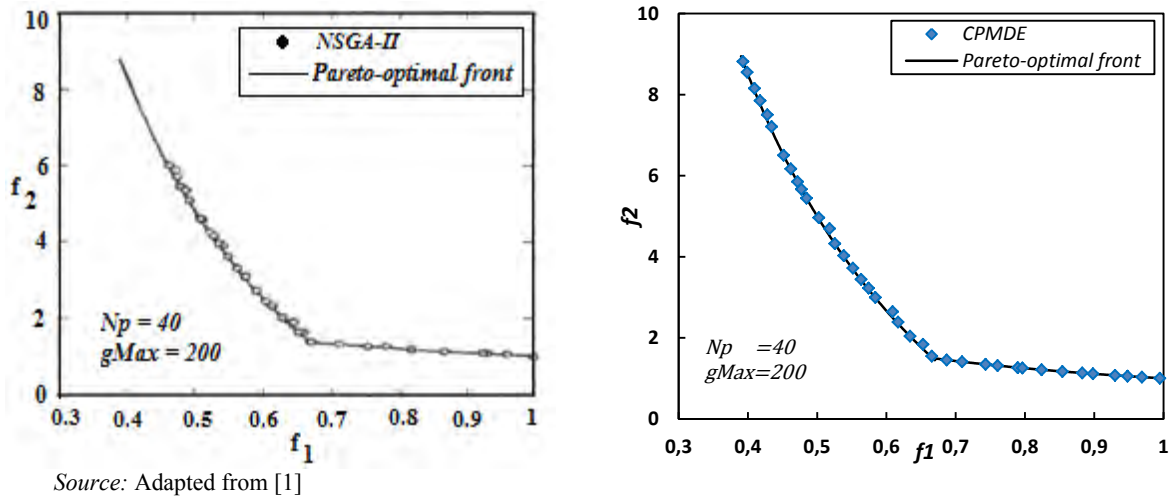


Figure 3: Performance of NSGA-II and CPMDE on test problem CONSTR, 200 iterations.

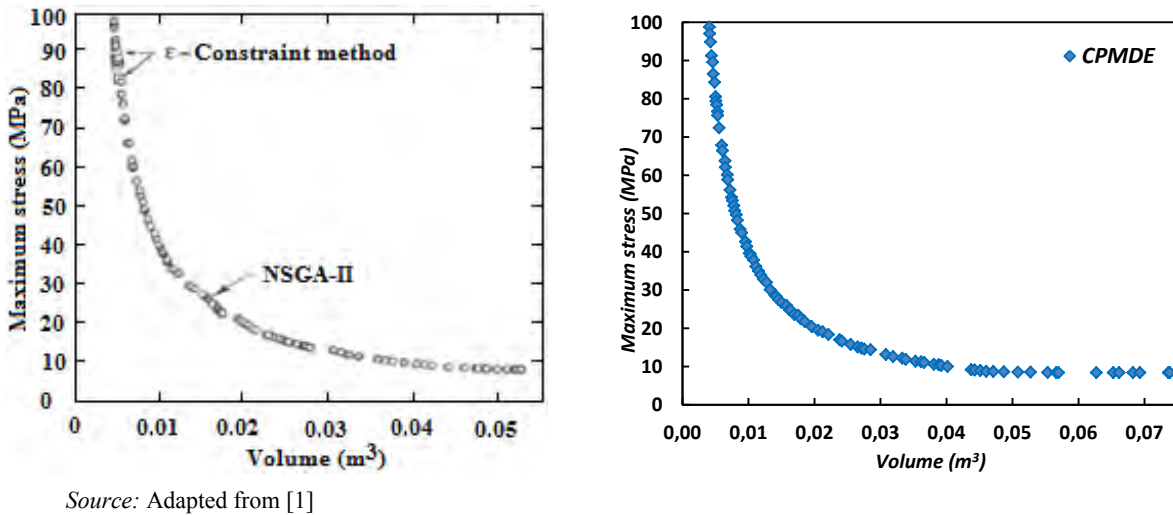


Figure 4: Performance of NSGA-II and CPMDE on a two-bar truss design problem.

## 5. CONCLUSION

A benchmark of Combined Pareto multi-objective differential evolution (CPMDE) on tuneable multi-objective test problems is presented in this study. The ability of CPMDE in solving constrained and real multi-objective optimization problems was also illustrated. The ability of CPMDE to converge to the global Pareto-optimal front in deceptive multi-modal functions is amply demonstrated on test problem ZDT4 which has 21 billion local optimal fronts. Among the 14 algorithms compared in this study, CPMDE produced the best convergence in 3 out of the 5, and best diversity in 4 out of 5 unconstrained test beds. Also, the variances of the metrics suggest that the algorithm is stable on the test beds. Furthermore, CPMDE was applied to solve a real-world problem where its efficacy on such problems was confirmed. Competitive results obtained from the application of CPMDE suggest that it is a good alternative for solving multi-objective optimization problems. Therefore, this study further corroborates that CPMDE is adoptable as a method of EMOA for solving real-world MOOPs.

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