MULTIPLE TRAFFIC JAMS IN FULL VELOCITY DIFFERENCE MODEL WITH REACTION-TIME DELAY

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Abstract
A full velocity difference model for traffic flow, including driver’s reaction time delay is considered. The uniform flow and traffic jams are interpreted through stability and bifurcation analysis. Specifically, the uniform flow is represented by the equilibrium of the model. Linear stability reveals that when the equilibrium loses its stability, local bifurcation turn up through Hopf bifurcations. To analyse the behaviour of the model after bifurcating, numerical continuation techniques are employed. Branches of oscillating solutions and the corresponding stabilities are obtained. It is shown that bifurcating oscillations can coexist and correspond to different traffic patterns. To visualize the spatial patterns, numerical simulations are performed, which are presented by velocity time histories and spatio-temporal diagrams. Analysing the characteristic features, these oscillating solutions are classified into three types, and further correspond to three types of traffic jams: almost traffic jams, width-equal traffic jam and width-alternated traffic jam. The obtained results provide an explanation of how multiple jams induced by driver’s reaction time delay occur.

Key Words: Full Velocity Difference Model, Reaction-Time Delay, Stability, Bifurcation, Numerical Continuation, Traffic Patterns

1. INTRODUCTION
In recent years, traffic flow problems have received considerable interest from various disciplines [1]. One of the reasons is that traffic jams are becoming more and more serious problems. However, although part of our daily lives, traffic dynamics and especially, traffic jams, have not been well understood. To study the traffic dynamics and to understand the mechanism of traffic jams, modelling of the traffic flow is the first step. In continuum models, instead of considering individual vehicles, continuous density distribution and velocity distribution (as functions of space and time) are used to describe the traffic flow, where partial differential equations (PDEs) are employed to describe the traffic flow, where partial differential equations (PDEs) are employed to describe the traffic flow [2]. In cellular automata models, space, time and velocity are considered to be discrete, and updated rules are used to describe the time development of traffic flow [3]. In car-following models, discrete entities move in continuous space and time, and the vehicles’ motions are described by ordinary differential equations (ODEs) [4] or by delay differential equations (DDEs) if time delay is introduced [5]. The optimal velocity (OV) model [4] is the most widely used car-following model. Generally, the quality of these developed models is evaluated by fitting the models to empirical data [6]. The data-fitting method easily leads to research capturing. However, some essential characteristics of traffic dynamics, e.g., the coexistence of multiple stable traffic flow, cannot be fully revealed through such data-fitting method. Hence, to comprehensively understand and classify the characteristic features of traffic flow, especially, the ‘hidden’ unstable motion, general models with varying traffic flow parameters need to be studied [7].

Among the previous research, bifurcation theory is a useful and effective way to study the dynamical behaviour of the traffic flow models. Igarashi et al. [8] first considered the bifurcation phenomena in a car-following model and pointed out that the subcritical Hopf bifurcation was the dynamical origin of the evolution between uniform flow and congested
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