SUPPLY CHAIN COORDINATION USING REVENUE-SHARING CONTRACT WITH DISTRIBUTOR’S EFFORT DEPENDENT DEMAND

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Abstract
Revenue-sharing contracts are a kind of mechanism aimed at improving performance and achieving precise coordination of the supply chain. In this paper, we analyse and develop the revenue-sharing contract model of the three-level supply chain with distributor’s sales effort dependent demand. The paper discusses the impacts of sales efforts on coordination of the supply chain and explains the reasons why traditional revenue-sharing contracts cannot coordinate the supply chain in this condition. In order to coordinate the supply chain, supposing the distributor bears the sales effort costs, the paper proposes an improved revenue-sharing contract based on a quantity discount policy. Three conditions are taken into consideration: the improved contract is only implemented between the retailer and the distributor, only implemented between the distributor and the manufacturer, and implemented both between the retailer and the distributor and between the distributor and the manufacturer. The paper shows that the improved revenue-sharing contract can coordinate the supply chain by carrying it out in one transaction or two transactions of the three-level supply chain. By supposing the effort and the market demand satisfy the multiplication form, we characterize the optimal decision variables (sales effort and inventory quantity). At the end, a numerical example is given to demonstrate the correctness of this paper.

Key Words: Supply Chain Coordination, Revenue-Sharing Contract, Effort, Quantity Discount

1. INTRODUCTION

With increasingly intense market competition, companies have realized that setting up a supply chain with other companies is an effective way to improve their core competitiveness. Usually, these various companies have independent decision-making power (decentralized decision-making). When supply chain members try to optimize their own profits, the whole profit of the supply chain will inevitably be affected, which is the so-called “double marginalization”. To deal with this problem, supply chain contracts are designed to facilitate interaction between the disparate supply chain members and to motivate participants to behave in the best interests of the supply chain (maximization of total supply chain profit). By specifying contract parameters such as order quantity, the supply chain contracts can provide protection for supply chain members against self-seeking behaviour [1]. Many contracts, including the wholesale price contract, the buy back contract, the revenue-sharing contract, the quantity flexibility contract and the option contract, have been applied to coordinate the supply chain.

The revenue-sharing contract [2], in which retailers pay royalties on product sales to suppliers, has been widely applied in supply chains, particularly in the video rental and movie industries. The contract can be described by two parameters ($w$, $\phi$): the supplier charges the retailers a unit wholesale price $w$, lower than the unit marginal cost $c$, in exchange for $(1 - \phi)$ percentage of the retailer’s revenue. The condition $w < c$ guarantees channel coordination whereas $\phi$ determines the distribution of total profits between the supplier and the retailer. In particular $\phi$ is the supply chain profit quota gained by the retailer. In recent years, supply
chains in various industries, especially in the movie industry, have adopted revenue sharing agreements [3].

Cachon [1, 2] pointed out that the revenue-sharing contract cannot coordinate the supply chain with effort dependent market demand. However, the effort activity of supply chain members is one of the important factors affecting market demand, such as increasing investment in advertising, hiring more sales staff to promote products, considering customers; personalities in product manufacturing, etc. [4]. The effort activities are beneficial to the entire supply chain system and could increase the market demand to some extent [5-7], but it also needs cost to do these effort activities. Therefore, if only one supply chain member bears the cost of effort activities, he will choose the effort level which is most beneficial to him, while this choice often cannot coordinate the supply chain [8]. These contradictions may be resolved when all supply chain members share the effort cost. But, in many cases, due to the fact that the effort of each supply chain member cannot be observed, all kinds of hidden ethical hazards will make the policy of sharing effort cost fail to coordinate the supply chain. As a result, it is not an effective way to coordinate the supply chain by sharing effort cost among supply chain members.

There is little literature that take the distributor’s effort activities into account when the coordination of supply chain is studied. In the available literature, the researchers often suppose that the retailer or the manufacturer implements the effort activities. The retailer or the manufacturer has some advantages to implement effort activities in the product processes, such as managing product sales layout, improving product quality, reducing production cost, designing personalized products, meeting the requirement of green environmental protection, and so on. While the distributor has his own unique advantage in market development, information communication and so on, there is no doubt that it will be highly beneficial to the supply chain if the distributor carries out more effort activities. So it is necessary to study how to coordinate the supply chain with distributor’s effort dependent demand.

In this paper, we extend the research object to a three-level supply chain which addresses the actual circumstances better and try to further study the following aspects. First, by analysing the behaviour of the distributor under a decentralized supply chain with the traditional revenue-sharing contract, we find the contract fails to coordinate the supply chain. Second, by supposing only the distributor bears the effort cost, we propose an improved revenue-sharing contract based on a quantity discount policy that can coordinate the three-level supply chain. As there are two different transaction phases and the distributor is in the middle of the three-level supply chain, three different conditions are discussed: the improved contract is only implemented between the retailer and the distributor, only implemented between the distributor and the manufacturer, and implemented the improved contract both between the retailer and the distributor and between the distributor and the manufacturer. Third, by supposing the effort and the market demand satisfy multiplication form, we characterize the optimal decision variables (sales effort and inventory quantity). Finally, a numerical example is given to illustrate the solution process of the model.

2. LITERATURE REVIEW

Revenue-sharing contract can coordinate the supply chain without effort dependent demand so that optimal supply chain performance can be achieved [1]. Successful applications of revenue-sharing contract in supply chain practice contribute to the growing of relevant researches to a large extent and the literature [9-17] is representative of this. After studying the literature of [2], Giannoccaro and Pontrandolfo [18] proposed a model based on revenue-sharing contract to coordinate a three-level supply chain. From the basis of [18], the model was further improved [19-21].
The revenue-sharing contract cannot coordinate supply chain with effort dependent demand. To solve this problem, scholars have looked into the revenue-sharing contract with effort dependent demand and determined that there are three main models which can be utilized as ways to coordinate supply chain: all supply chain members share the effort cost, only the retailer bears the effort cost and only the manufacturer bears the effort cost.

Considering the first model, all supply chain members share the effort cost, Hu and Wang [22] analyse retailer’s effort level’s impact on supply chain revenue-sharing contract and determine the retailer’s effort level’s reaction function, then obtain the optimal revenue-sharing coefficient and cost-sharing coefficient by which the revenue-sharing evolution contract can realize supply chain coordination. Dai and Meng [23] investigate the impacts of the retailer’s promotional effort and risk aversion on the feasible domain of revenue-sharing contract. At the same time, they point out that both the sales effect and risk aversion reduce the feasible range of the efficient contract. In addition, the introduction of the cost-sharing mechanism helps eliminate the adverse impact of the promotional effect on the contract’s feasible region. Sun et al. [24] analyses the influence of the proportion that the retailer shares in the effort cost with two manufacturers on the product demand, and designs an improved revenue-sharing contract to achieve the supply chain coordination. Qu and Guo [25] develop a two-level supply chain which has hybrid distribution channels, and proposed an improved revenue-sharing contract which can coordinate the supply chain by having the retailer and the manufacturer share the effort cost.

Considering the second model, only the retailer bears the effort cost, He et al. [26, 27] conceived of an augmented revenue-sharing contract based on rebate & penalty contract to coordinate the supply chain with demand dependent effort, in which collaboration and a win-win outcome are achieved. Han et al. [28] studies the revenue sharing contract with effort dependent demand based on exponential function for price, and designs an improved revenue-sharing contract on the basis of a reward and penalty policy to coordinate the supply chain. Pang et al. [29] also proposes an improved revenue-sharing contract based on a rebate and penalty policy, but it is used to coordinate the three-level supply chain by implementing it in one transaction or two transactions of the three-level supply chain. They point out that the former is a special case of the latter.

Considering the third model, only the manufacturer bears the effort cost, Chen and Chen [30] study the application of revenue-sharing contract in the virtual enterprises with market demand dependent effort. Pang et al. [31] propose the introduction of the quantity discount policy in one transaction or two transactions of the three-level supply chain to solve the problem of the revenue-sharing contract being unable to coordinate supply chain. From the basis of Pang et al. [29], the model is further improved [32].

With the development of society, the supply chain is becoming more and more dependent on the role that the distributor plays, who is in the middle of the supply chain. The distributor has a unique advantage in developing the service channel, integrating social resources, and optimizing the inventory etc. However, after searching, we have not seen any literature that takes the distributor’s efforts in three-level supply chain into account.

The main contribution of this paper is that we consider the distributor’s effort in the coordination of three-level supply chain and propose an improved revenue-sharing contract based on a quantity discount policy, which can coordinate the three-level supply chain. Therefore, our objective in this paper is to improve the revenue-sharing contract to coordinate a three-level supply chain where the distributor bears the effort cost. The questions that we emphasize in this paper are as follows. What are the impacts of the distributor’s effort on supply chain coordination? Can the supply chain be coordinated by using revenue-sharing contracts? If not, what should we do to improve and implement revenue-sharing contracts in the supply chain?
3. ANALYTICAL MODEL

3.1 Model description

The supply chain studied in this paper is made up of one manufacturer (m), one distributor (d) and one retailer (r). The upstream member provides a single product to the downstream member, and the demand is uncertain. Before sale season, the distributor and the retailer have only one chance to buy products. All members are risk neutral, and information is symmetric among them.

Suppose \( p \) is sales price of unit product; \( c_i \) is supply chain member’s marginal unit costs \((i = m, d, r)\) and \( c = c_r + c_d + c_m; \) \( v \) is salvage value for unsold unit product \((v < c)\); \( w_i \) is the wholesale price that upstream member charges downstream member in \( j \) condition \((j = c, d; i = m, d)\); \( e \) is the distributor’s effort, and \( e \geq 0 \); \( g(e) \) is the distributor’s effort cost when his effort is \( e \), supposing \( g(0) = 0 \), and \( g'(e) > 0 \) when \( e > 0 \); \( x \) is the stochastic demand when the distributor’s effort is \( e \), with probability density function \( f(x|e) \) and differentiable cumulated distribution function \( F(x|e) \), and \( F(x|e) \) is continuously differentiable. Because the demand is the increasing function of effort, \( \partial F(x|e)/\partial e < 0; Q \) is the order quantity; when the distributor’s effort is \( e \), the expectation of sale quantity is \( S(Q, e) = E_{min}(Q, x) = Q - \int_{0}^{\infty} F(x|e)dx \), and \( \partial S(Q, e)/\partial e > 0 \); the expectation of unsold quantity is \( I(Q, e) \) and \( I(Q, e) = Q - S(Q, e) \). In particular, it is reasonable that \( w_{d} > w_{m} > c_{d}, v < c < p, w_{m} + c_{r} < p \).

3.2 Analysis of revenue-sharing contract

Supply chain contracts often apply incentive measures to adjust the relationship among the members to coordinate the supply chain, and to make the entire profit of the decentralized supply chain equal to that of the centralized supply chain as much as possible. The goal of the analysis is to design the revenue-sharing contract so as to achieve channel coordination (maximum profit for the whole supply chain). In addition, it aims at analysing whether the contract parameters can be modified and how to modify the parameters so as to more evenly share the profit along the supply chain, a win-win situation for the chain partners, guaranteeing channel coordination. So, the maximum profit of the centralized supply chain should be considered as the primary objective of revenue-sharing contract to coordinate the supply chain. The profit function of the centralized supply chain can be described as eq. (1).

\[
\Pi(Q, e) = pS(Q, e) + vI(Q, e) - cQ - g(e) = (p - v)S(Q, e) - (c - v)Q - g(e)
\]

(1)

Supposing \( e^* \) is the distributor’s optimal effort when given order quantity \( Q \), then \( e^* \) should satisfy eq. (2).

\[
\partial \Pi(Q, e^*)/\partial e = (p - v)\partial S(Q, e^*)/\partial e - g'(e^*) = 0
\]

(2)

Supposing \( Q^* \) is the retailer’s optimal order quantity when given effort \( e \), then \( Q^* \) should satisfy \( \partial \Pi(Q^*, e)/\partial Q = (p - v)\partial S(Q^*, e)/\partial Q - (c - v) = 0 \) . Namely \( Q^* \) should satisfy eq. (3):

\[
F(Q^*|e) = \frac{p - c}{p - v}
\]

(3)

Therefore, if the revenue-sharing contract can coordinate the supply chain, then eqs. (2) and (3) are the necessary conditions to achieve supply chain coordination.

Giannoccaro and Pontrandolfo [18] proposed a model of the revenue-sharing contract to coordinate three-level supply chain without taking the supply chain member’s stockout loss and salvage value of unsold unit product into account. In this paper, we analyse whether the revenue-sharing contract can coordinate a three-level supply chain considering salvage value of unsold unit product and the retailer’s effort. Supposing the retailer would keep a quota \( \phi_2 \).
of his revenue, give the rest \((1 - \phi_2)\) to the distributor and this would be balanced by a lower price \(w^c_d\). Similarly, the distributor would keep a quota \(\phi_1\) of his revenue, give the rest \((1 - \phi_1)\) to the manufacturer and this would be balanced by a lower price \(w^c_m\). Also assume only the distributor bears the effort cost. Then the expected profit function of the retailer, the distributor and the manufacturer can be written as the following:

\[
\Pi_r(Q,e) = \phi_1 \left[pS(Q,e) + vI(Q,e)\right] - c_rQ - w^c_dQ = \phi_1 \left[p - v\right]S(Q,e) - \left(c_r + w^c_d - \phi_2 v\right)Q
\]

(4)

\[
\Pi_d(Q,e) = \phi_1 \left[(1 - \phi_2) \left(pS(Q,e) + vI(Q,e)\right) + w^c_dQ\right] - c_dQ - w^c_mQ - g(e)
\]

(5)

\[
\Pi_m(Q,e) = (1 - \phi_1) \left[(1 - \phi_2) \left(pS(Q,e) + vI(Q,e)\right) + w^c_dQ\right] + w^c_mQ - c_mQ
\]

(6)

From eq. (5), we get \(\frac{\partial \Pi_d(Q,e)}{\partial e} = -g'(e) + (1 - \phi_1)(1 - \phi_2)(p - v)\frac{\partial S(Q,e)}{\partial e}\). Compared with eq. (2), it can be seen that the distributor’s effort \(e\) in this condition is smaller than the optimal effort of the centralized supply chain, namely, \(\frac{\partial \Pi_d(Q,e)}{\partial e} < \frac{\partial \Pi_c(Q,e)}{\partial e}\). That is to say, under this condition, the revenue-sharing contract cannot coordinate the supply chain. The reason lies in the fact that the distributor bears all the effort cost of the supply chain, but he only gets partial profit of the whole supply chain. Similarly, if only the retailer or only the manufacturer bears the effort cost, the traditional revenue-sharing contract cannot coordinate the supply chain [32]. Thus we come to a conclusion: the traditional revenue-sharing contract cannot coordinate a three-level supply chain with effort dependent demand.

In this paper, an improved revenue-sharing contract based on a quantity discount policy is proposed to coordinate the three-level supply chain with demand dependent effort.

4. AN IMPROVED REVENUE-SHARING CONTRACT BASED ON QUANTITY DISCOUNT POLICY

From the discussion above, we know that the optimal effort of the supply chain member under revenue-sharing contract is not equal to that of the centralized supply chain. The reason lies in that the supply chain member only gets partial gains but bears all the effort cost \(g(e)\) of the whole supply chain. To solve this problem, we can make his gains equal to a fixed ratio of the gains of the whole supply chain. Here, we propose an improved revenue-sharing contract based on quantity discount policy to coordinate the supply chain. Because there are two different transaction phases, up-stream process and downstream process, and the distributor is in the middle of the supply chain, three situations are needed to be considered: implement only between the distributor and the retailer, implement only between the manufacturer and the distributor, and implement among all the supply chain members.

4.1 Implementing only between distributor and retailer

Under the first model, based on a policy of quantity, we implement the revenue-sharing contract between the distributor and the retailer, and implement the traditional revenue-sharing contract between the manufacturer and the distributor. That is, the distributor provides the retailer the revenue-sharing contract \((\phi_2, w^c_d(\cdot))\), and the manufacturer provides the distributor the revenue-sharing contract \((\phi_1, w^c_m)\). Supposing:

\[
w^c_d(Q) = \phi_2 c_r - c_r + \left[\frac{1}{\phi_1} - (1 - \phi_2)\right] \frac{g(e)}{Q}
\]

(7)
Take eq. (7) into the distributor’s profit function eq. (5), and we can get:
\[ \Pi_d^d(Q,e) = \phi_1 (1-\phi_2) \left( pS(Q,e) + v(L(Q,e)) + w_d^d(Q)Q \right) - c_dQ - w_m^dQ - g(e) \]
\[ = \phi_1 (1-\phi_2) \left( p-v \right) S(Q,e) - \left[ c_d + w_m^d - \phi_1 (1-\phi_2) \right] Q - \phi_1 (1-\phi_2) g(e) \] (8)

According to the paper [18], without considering demand dependent effort, the ratio that the distributor’s profit makes up of the profit of the supply chain is \( \phi_1 (1-\phi_2) \). Here, supposing only the distributor bears the effort cost, the ratio that his profit shares the profit of the whole supply chain also is \( \phi_1 (1-\phi_2) \). It can be easily seen that the effort of the distributor would be equal to the optimal effort of the supply chain, that is,
\[ \Pi^*_d(Q,e) = \phi_1 (1-\phi_2) \Pi^*_d(Q,e) \] (9)

From eq. (9), we can get:
\[ w_m^d = \phi_1 c - \phi_1 c_r - c_d \] (10)

Take eqs. (7) and (10) into the eqs. (4) and (6), and we get:
\[ \Pi^*_m(Q,e) = \phi_2 \left( p-v \right) S(Q,e) - \phi_2 \left( e-v \right) Q - \left[ 1 - \frac{\phi_1}{\phi_1} \right] g(e) \] (11)

\[ \Pi^*_m(Q,e) = (1-\phi_1) \left( 1-\phi_2 \right) \left( p-v \right) S(Q,e) + w_d^* Q + w_m^* Q - c_m Q \]
\[ = (1-\phi_1) \left( 1-\phi_2 \right) \left( p-v \right) S(Q,e) - \left[ c_m - w_m^* - (1-\phi_1) \right] (1-\phi_2) v - (1-\phi_1) w_d^* Q \] (12)

Here \( e^* \) is the optimal effort of the supply chain.

From eqs. (9), (11) and (12), it can be seen that
\[ \frac{\partial \Pi^*_m(Q,e)}{\partial Q} = \frac{\partial \Pi^*_m(Q^*,e)}{\partial Q} = \frac{\partial \Pi^*_m(Q',e)}{\partial Q} \]

This means the optimal order (production) quantity of the supply chain member is equal to the optimal order quantity of the centralized supply chain.

From eqs. (9), (11) and (12), it can also be seen that the profit functions of the supply chain members are all affine functions of the whole supply chain’s profit function. So in this situation, the revenue-sharing contract can coordinate the three-level supply chain.

Therefore, if we implement the revenue-sharing contract based on quantity discount policy between the retailer and the distributor and implement the traditional revenue-sharing contract between the distributor and the manufacturer, and the contract parameters satisfy eq. (7) and eq. (10), the improved revenue-sharing can coordinate the three-level supply chain.

Compared with the model of Giannoccaro and Pontrandolfo [18], the profit of the retailer decreases by \( \left( \frac{\phi_1}{\phi_1} - 1 \right) g(e^*) \), while the profit of the manufacturer increases by \( \left( \frac{\phi_1}{\phi_1} - 1 \right) g(e^*) \).

4.2 Implementing only between manufacturer and distributor

Under the second model, based on a policy of quantity, we implement the revenue-sharing contract between the manufacturer and the distributor, and implement the traditional revenue-
sharing contract between the retailer and the distributor. Namely, the distributor provides the retailer the revenue-sharing contract \( (\phi_r, w_r^e) \), and the manufacturer provides the distributor the revenue-sharing contract \( (\phi_d, w_d^e(Q)) \). Supposing:

\[
w_d^e(Q) = \phi_d - [1 - \phi_r(1 - \phi_d)] g(e) - \phi_d c_r + c_d \tag{13}
\]

Take eq. (13) into the distributor’s profit function eq. (5), and we can get:

\[
\Pi_d(Q,e) = \phi_d \left( (1 - \phi_r)(p S(Q,e) + v I(Q,e)) + w_d^e \right) - c_d Q - c_r - w_d^e - \phi_d (1 - \phi_r) g(e)
\]

\[
= \phi_d (1 - \phi_r) S(Q,e) - \phi_r \left[ c r - w_d^e - (1 - \phi_r) v \right] Q - \phi_d (1 - \phi_r) g(e)
\]

\[
= \phi_d (1 - \phi_r) \Pi_r(Q,e) \tag{15}
\]

Similarly, supposing only the distributor bears the effort cost, the ratio that his profit shares the profit of the whole supply chain also is \( (\phi_r, w_r^e) \). It is easily seen that the effort of the distributor would be equal to the optimal effort of the supply chain, that is,

\[
\Pi_d(Q,e) = \phi_r (1 - \phi_r)(p S(Q,e) + v I(Q,e)) - \phi_r \left[ c r - w_d^e - (1 - \phi_r) v \right] Q - \phi_r (1 - \phi_r) g(e)
\]

From eq. (15), we can get:

\[
w_d^e = \phi_d c_r - c_r \tag{16}
\]

Take eqs. (13) and (16) into the eqs. (4) and (6), and we can get:

\[
\Pi_r(Q,e) = \phi_r \left[ p S(Q,e) + v I(Q,e) \right] - c_r Q - w_d^e Q = \phi_r (p - v) S(Q,e) - \phi_r (c - v) Q
\]

\[
= \phi_r \Pi_i(Q,e) + \phi_r g(e^*) \tag{17}
\]

\[
\Pi_m(Q,e) = (1 - \phi_r) \left[ (1 - \phi_d) (p S(Q,e) + v I(Q,e)) + w_d^e \right] - c_m Q
\]

\[
= (1 - \phi_r)(1 - \phi_d) (p - v) S(Q,e) - (1 - \phi_r) (1 - \phi_d) (c - v) Q
\]

\[
= (1 - \phi_r)(1 - \phi_d) \Pi_i(Q,e) - \phi_r g(e^*) \tag{18}
\]

Here \( e^* \) is the optimal effort of the supply chain.

From eqs. (15), (17) and (18), it also can be seen that \( \frac{\partial \Pi_r(Q^*,e)}{\partial Q} = \frac{\partial \Pi_i(Q^*,e)}{\partial Q} = \frac{\partial \Pi_m(Q^*,e)}{\partial Q} = \frac{\partial \Pi_d(Q^*,e)}{\partial Q} \). This means the optimal order (production) quantity of the supply chain member is equal to the optimal order quantity of the centralized supply chain.

From eqs. (15), (17) and (18), it also can be seen that the profit functions of the supply chain members are all affine functions of the whole supply chain’s profit function. So in this situation, the revenue-sharing contract can coordinate the three-level supply chain.

Therefore, if we implement the revenue-sharing contract based on quantity discount policy between the distributor and the manufacturer and implement the traditional revenue-sharing contract between the retailer and the distributor, while the contract parameters satisfy eqs. (13) and (16), the improved revenue-sharing can coordinate the three-level supply chain.

Compared with the model of Giannoccaro and Pontrandolfo [18], the profit of the retailer increases by \( \phi_r g(e^*) \), while the profit of the manufacturer decreases by \( \phi_d g(e^*) \).

### 4.3 Implementing among all supply chain members

Under the third model, based on a policy of quantity, we implement the revenue-sharing contract not only between the manufacturer and the distributor, but also between the retailer and the distributor. That is, the distributor provides the retailer the revenue-sharing contract
(\phi, w^e_d(O))$, and the manufacturer provides the distributor the revenue-sharing contract $(\phi, w^e_m(O))$. Supposing:

$$w^e_d(O) = \phi_2c + (1-\phi_2)\frac{g(e)}{Q} - c_r$$

(19)

$$w^e_m(O) = \phi_1c + \frac{g(e)}{Q} - \phi_1c_r - c_d$$

(20)

Take eqs. (19) and (20) into the distributor’s profit function eq. (5), and we can get:

$$\Pi^e_d(O, e) = \phi_1[(1-\phi_2)(\rho S(O, e) + v^f(O, e)) + w^e_d(O)Q] - c_r Q - w^e_m(O)Q - g(e)$$

$$= \phi_1(1-\phi_2)(p-v)S(O, e) - \phi_1(1-\phi_2)(c-v)Q - \phi_1(1-\phi_2)g(e)$$

(21)

$$= \phi_1(1-\phi_2)\Pi_d(O, e)$$

From eq. (21), it can be seen that the optimal effort of the distributor is equal to the optimal effort of the supply chain.

Take eqs. (19) and (20) into the eqs. (4) and (6), so we can get:

$$\Pi^e_m(O, e) = \phi_2[pS(O, e) + v^f(O, e)] - c_r Q - w^e_d(O)Q$$

$$= \phi_2(p-v)S(O, e) - \phi_2(c-v)Q - (1-\phi_2)g(e)$$

(22)

$$= \phi_2\Pi_d(O, e) - (1-2\phi_2)g(e^*)$$

$$\Pi^e_m(O, e) = (1-\phi_1)[(1-\phi_2)(\rho S(O, e) + v^f(O, e)) + w^e_d(O)Q] + w^e_m(O)Q - c_m Q$$

$$= (1-\phi_1)(1-\phi_2)(p-v)S(O, e) - (1-\phi_1)(1-\phi_2)(c-v)Q - \phi_1\phi_2 g(e)$$

(23)

$$= (1-\phi_1)(1-\phi_2)\Pi_m(O, e) + (1-2\phi_2)g(e^*)$$

Here $e^*$ is the optimal effort of the supply chain.

From eqs. (21), (22) and (23), it can also be seen that:

$$\frac{\partial \Pi^e_m(O^*, e)}{\partial Q} = \frac{\partial \Pi^e_d(O^*, e)}{\partial Q} = \frac{\partial \Pi^e_m(O^*, e)}{\partial Q} = \frac{\partial \Pi^e_m(O^*, e)}{\partial Q}$$

This means the optimal order (production) quantity of the supply chain member is equal to the optimal order quantity of the centralized supply chain.

From eqs. (21), (22) and (23), it also can be seen that the profit functions of the supply chain members are all affine functions of the whole supply chain’s profit function. So in this condition, the revenue-sharing contract can coordinate the three-level supply chain.

Therefore, if we implement the revenue-sharing contract based on a policy of quantity discount among all the supply chain members, and the contract parameters satisfy eqs. (19) and (20), the improved revenue-sharing can coordinate the three-level supply chain.

Compared with the model of Giannoccaro and Pontrandolfo [18], the profit of the retailer decreases by $(1-2\phi_2)g(e^*)$, while the profit of the manufacturer increases by $(1-2\phi_2)g(e^*)$.

It needs to be pointed out that the expression of $w^e_d(O)$ and $w^e_m(O)$ are not the only form. The forms such as eqs. (19) and (20) are more common. Further, if we let $e = 0$, that is the distributor does not carry out any effort activities, all the forms of $w^e_d(O)$ and $w^e_m(O)$ in this section can be written as $w^e_d = \phi_2c - c_r$ and $w^e_m = \phi_1c - \phi_1c_r - c_d$, which also confirms the validity of this paper’s proposals.

5. COMPARISON OF THREE CONDITIONS

From the elaboration above, we can get Table I.
Here, A means that we implement the revenue-sharing contract based on quantity discount policy only between the retailer and the distributor. B means that we implement the revenue-sharing contract based on quantity discount policy only between the distributor and the manufacturer. C means that we implement the revenue-sharing contract based on quantity discount policy in two transactions of the supply chain.

Table I indicates that the ratio that the distributor shares the profit of the whole supply chain is \( \phi \left(1 - \phi_2\right) \), which guarantees that the effort of the distributor would be equal to the optimal effort of the supply chain. When the profit of the supply chain is a fixed value, which means the profit of the distributor is also a fixed value, the profit of the retailer and the profit of the manufacturer will adjust in the opposite direction. That is, if the profit of the retailer is the maximum value in A (B or C) condition, the profit of the manufacturer will be the minimum value in A (B or C) condition. So we only need to analyse under which condition the retailer prefers to implement the improved revenue-sharing contract.

Due to \( 0 < \phi_2 < 1 \), so \( \phi_2 > 2 \phi_2 - 1 \), which means the retailer will share more profit of the supply chain in condition B than in condition C. That is, B is always better than C for the retailer. If \( (1 - 1/\phi_2) \geq \phi_2 \), the optimal selecting sequence for the retailer is (A, B, C). If \( \phi_2 > 1 - 1/\phi_2 > 2 \phi_2 - 1 \), the optimal selecting sequence for the retailer is (B, A, C). If \( 2 \phi_2 - 1 > 1 - 1/\phi_2 \), the optimal selecting sequence for the retailer is (B, C, A).

Both this paper and the literature [32] study the profit of the supply chain members when only one member bears the effort cost in condition C. Table II shows the profit of the supply chain members in condition C.

<table>
<thead>
<tr>
<th>Table I: Comparison of three conditions.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>( \Pi^*_r )</td>
</tr>
<tr>
<td>( \Pi^*_d )</td>
</tr>
<tr>
<td>( \Pi^*_m )</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Table II: Comparison of the profit of the supply chain members in condition C.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Literature [32]</td>
</tr>
<tr>
<td>( \Pi^*_r )</td>
</tr>
<tr>
<td>( \Pi^*_d )</td>
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<tr>
<td>( \Pi^*_m )</td>
</tr>
<tr>
<td>This paper</td>
</tr>
<tr>
<td>( \Pi^*_r )</td>
</tr>
<tr>
<td>( \Pi^*_d )</td>
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<tr>
<td>( \Pi^*_m )</td>
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</tbody>
</table>

In table II, R means only the retailer bears the effort cost. M means only the manufacturer bears the effort cost. D means only the distributor bears the effort cost.

From table II, we know that the distributor shares the minimum value of the profit of the supply chain in condition M. If \( \phi_1 (1 - \phi_2) > 2 \phi_2 - 1 \), the profit of supply chain that the distributor shares in condition R is more than that the retailers shares in condition D. So if
\( \phi_1 (1 - \phi_2) > 2 \phi_2 - 1 \), the best sequence for the distributor to let the supply chain member bear the effort cost is (retailer, distributor, manufacturer); if \( \phi_1 (1 - \phi_2) \leq 2 \phi_2 - 1 \), the best sequence for the distributor to let the supply chain member bear the effort cost is (distributor, retailer, manufacturer).

Due to \( 0 < \phi_2 < 1 \), so \( 2 \phi_2 - 1 < 1 \), which means the retailer shares the maximum value of the profit of the supply chain in condition M. If \( 0.5 < \phi_2 < 1 \), the profit of supply chain that the retailer shares in condition D is more than that which the retailers shares in condition R. So if \( 0.5 < \phi_2 < 1 \), the best sequence for the retailer to let the supply chain member bear the effort cost is (manufacturer, distributor, retailer); if \( 0 < \phi_2 \leq 0.5 \), the best sequence for the retailer to let the supply chain member bear the effort cost is (manufacturer, retailer, distributor).

Similarly, due to \( 0 < \phi_2 < 1 \) and \( 0 < \phi_3 < 1 \), the profit of supply chain that the manufacturer shares in condition M is more than that which the manufacturer shares in condition R. If \( 0 < \phi_3 \leq 0.5 \), the best sequence for the manufacturer to let the supply chain member bear the effort cost is (distributor, manufacturer, retailer). If \( 0.5 < \phi_3 < 1 \) and \( \phi_1 (1 - \phi_2) > 2 \phi_2 - 1 \), the best sequence for the manufacturer to let the supply chain member bear the effort cost is (manufacturer, distributor, retailer). If \( 0.5 < \phi_3 < 1 \) and \( \phi_1 (1 - \phi_2) < 2 \phi_2 - 1 \), the best sequence for the manufacturer to let the supply chain member bear the effort cost is (manufacturer, retailer, distributor).

### 6. MODEL OPTIMIZATION

Suppose the market demand \( X(e, \xi) \) is the function of effort \( e \) and random factor \( \xi \), and \( \xi \) is independent of \( e \); \( f(\xi) \) and \( F(\xi) \) are respectively the probability density function and the differentiable cumulated distribution function of \( \xi \). Here suppose \( X(e, \xi) = y(e) \cdot \xi \). Because the influence of effort on marker demand is diminishing marginal utility, we can suppose \( y(e) \) is the monotonically increasing and concave function of effort \( e \), namely \( y'(e) > 0, y''(e) \leq 0 \).

When the market demand satisfies \( X(e, \xi) = y(e) \cdot \xi \), we can get:

\[
S(Q, e) = Q - \int_0^Q F(|x|) dx = Q - \int_0^Q F(|y(e)|) dx = Q - y(e) \int_0^Q F(t) dt
\]

(24)

So the profit function of supply chain can be rewritten as:

\[
\Pi_t(Q, e) = (p - v) S(Q, e) - (c - v)Q - g(e) = (p - c)Q - (p - v) y(e) \int_0^Q F(t) dt - g(e)
\]

(25)

Given effort \( e \), it can be seen that \( \Pi_t(Q, e) \) is the concave function of order quantity \( Q \). So the optimal order quantity \( Q^* \) should satisfy eq. (26):

\[
\frac{\partial \Pi_t(Q^*, e)}{\partial Q} = (p - c) - (p - v) F\left(Q^*/y(e)\right) = 0
\]

(26)

From eq. (26), we can get:

\[
Q^* = Q'(e) = y(e) F^{-1} \left( \frac{p - c}{p - v} \right)
\]

(27)

Then the profit function of supply chain can be rewritten as eq. (28):

\[
\Pi_t(Q^*, e) = (p - c) y(e) F^{-1} \left( \frac{p - c}{p - v} \right) - (p - v) y(e) \int_0^Q F\left(\frac{p - c}{p - v}t\right) dt - g(e) = (p - v) y(e) \int_0^Q F\left(\frac{p - c}{p - v}t\right) dt - g(e)
\]

(28)

According to the assumption, \( \Pi_t(Q^*, e) \) is the concave function of effort \( e \). So the optimal effort \( e^* \) of the supply chain should satisfy eq. (29):

\[
\frac{\partial \Pi_t(Q^*, e^*)}{\partial e} = (p - v) y'(e^*) \int_0^Q F\left(\frac{p - c}{p - v}t\right) dt - g'(e^*) = 0
\]

(29)
From eq. (29), it can be easily determined that the optimal effort \( e^* \) of the supply chain should satisfy eq. (30).

\[
(p - v) y'(e^*) \int_0^{F^{-1}(p/c)} f(t) dt = g'(e^*)
\]

(30)

7. NUMERICAL EXAMPLE

Suppose a supply chain is made up of one retailer, one distributor and one manufacturer. The parameters of the supply chain are as follows: \( c_r = 1 \), \( c_d = 2 \), \( c_m = 5 \), \( v = 3 \), \( w_d = 18 \), \( w_m = 10 \), \( p = 35 \), \( \hat{\phi} = \phi_2 = 0.55 \). In order to facilitate the analysis, here we adopt the method of hypothesis in [7] to simplify the formulas. Suppose \( x = e\hat{\xi} \) and \( g(e) = ae^2 / 2 \), \( \hat{\xi} \) satisfies uniform distribution in \([60, 100]\). It is easy to get: \( f(\hat{\xi}) = 1/40 \), \( F(\hat{\xi}) = (\hat{\xi} - 60)/40 \), \( F^{-1}(\hat{\xi}) = 60 + 40\hat{\xi} \). Here, \( a = 100 \).

(1) The decentralized supply chain (The wholesale price contract)

“Double marginalization” will occur in the decentralized supply chain when every supply chain member wants to maximize his own profit. In this condition, the optimal effort of the distributor, the optimal order quantity of the retailer, the profit of the whole supply chain and the supply chain member are respectively:

\[
\begin{align*}
    & e^d = 4.13, \quad Q^d = 296, \quad \Pi_d(Q^d, e^d) = 6544.36, \quad \Pi_e(Q^d, e^d) = 683.54, \quad \Pi_m(Q^d, e^d) = 2983.48, \\
    & \Pi_m(Q^d, e^d) = 2877.34.
\end{align*}
\]

(2) The traditional revenue-sharing contract

In this condition, the optimal effort of the distributor, the optimal order quantity of the retailer, the profit of the whole supply chain and the supply chain member are respectively:

\[
\begin{align*}
    & e^d = 4.86, \quad Q^d = 411, \quad \Pi_d(Q^d, e^d) = 8237.51, \quad \Pi_e(Q^d, e^d) = 796.15, \quad \Pi_m(Q^d, e^d) = 3695.82, \\
    & \Pi_m(Q^d, e^d) = 3754.54.
\end{align*}
\]

(3) The revenue-sharing contract based on quantity discount policy

From the discussion above, we know that the revenue-sharing contract based on quantity discount policy can coordinate the three-level supply chain whether it is implemented in one or two transactions. According to the formulas mentioned above, it is easy to get the optimal effort of the distributor, the optimal order quantity of the retailer and the profit of the whole supply chain. Here, \( e^* = 13.72 \), \( Q^* = 762 \), \( \Pi_e(Q^*, e^*) = 15486.64 \).

Table III sets the parameters of the supply chain in different modes. Here, the meaning of A, B and C in Table I is the same as that in Table III.

Table III shows that the distributor’s effort in the revenue-sharing contract based on quantity discount policy is highest among the three modes, which can maximize the profit of the supply chain. In addition, the revenue-sharing contract based on quantity discount policy is better than the traditional revenue-sharing contract with effort dependent demand. The distributor’s effort in the traditional revenue-sharing contract is higher than that of the supply chain in the wholesale price contract, and the retailer’s order quantity is still higher than that in wholesale price contract due to the incentive of the revenue-sharing contract. So the profit of the supply chain in the traditional revenue-sharing contract is still higher than that in wholesale price contract. As a result, it can be seen that the traditional revenue-sharing contract is better than the wholesale price contract with effort dependent demand.

In practice, however, if the supply chain members are willing to accept the revenue-sharing contract to coordinate the supply chain, a necessary condition is that their profit in the revenue-sharing contract cannot be less than that in the decentralized supply chain. That is, \( \Pi_e(Q, e) \geq \Pi_e^d(Q, e) \), \( \Pi_d(Q, e) \geq \Pi_d^d(Q, e) \), \( \Pi_m(Q, e) \geq \Pi_m^d(Q, e) \). From Table III, we can find that
the manufacturer’s profit in B mode is lower than his profit in the decentralized supply chain, which means that although the supply chain can achieve coordination in mode B, it cannot be implemented in reality.

Table III: The parameters of supply chain in different modes with the distributor bearing effort cost.

<table>
<thead>
<tr>
<th></th>
<th>Revenue-sharing contract based on quantity discount policy</th>
<th>Traditional revenue-sharing contract</th>
<th>Wholesale price contract</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>Optimal order quantity $Q$</td>
<td>762</td>
<td>762</td>
<td>762</td>
</tr>
<tr>
<td>Profit of the retailer $\pi_r$</td>
<td>816.99</td>
<td>13694.21</td>
<td>7576.46</td>
</tr>
<tr>
<td>Profit of the distributor $\pi_d$</td>
<td>3832.94</td>
<td>3832.94</td>
<td>3832.94</td>
</tr>
<tr>
<td>Profit of the manufacturer $\pi_m$</td>
<td>10836.71</td>
<td>-2040.51</td>
<td>4077.24</td>
</tr>
<tr>
<td>Profit of the whole supply chain $\pi_t$</td>
<td>15486.64</td>
<td>15486.64</td>
<td>15486.64</td>
</tr>
</tbody>
</table>

8. CONCLUSIONS

With increasingly intense market competition, the position of distributor is becoming increasingly critical to supply chain coordination. Because the distributor is in the middle of the three-level supply chain, he has the unique advantage to carry out effort activities, such as market development, information communication and so on. It is clear that if the distributor carries out more effort activities, it will be highly beneficial to the whole supply chain. However, there is limited literature that takes the distributor’s effort activities into account when the coordination of the supply chain is studied. In the available literature, researchers often suppose that the retailer or the manufacturer implements the effort activities.

The main contribution of this paper is that we consider the distributor’s effort in the coordination of the three-level supply chain and propose an improved revenue-sharing contract based on a quantity discount policy, which can coordinate the three-level supply chain. In this paper, three conditions are taken into consideration: the improved contract is only implemented between the retailer and the distributor, only implemented between the distributor and the manufacturer, and implemented both between the retailer and the distributor and between the distributor and the manufacturer. The paper shows that the improved revenue-sharing can coordinate the three-level supply chain in all three conditions. The profits of the supply chain members in three conditions are compared and the optimal selecting sequence for the supply chain members is given. Also when the improved revenue-sharing contract is implemented among supply chain members and only one supply chain member bears the effort cost, the profits of the supply chain members are compared and the best sequence for the distributor (retailer or manufacturer) to let the supply chain member bear the effort cost is given. The paper also provides the method for determining optimal order quantity and optimal effort, which can provide the decision makers of the supply chain with support for making decisions.

Future research on the appropriateness and effects of supply chain member’s effort activities is warranted. Future research should consider a three-level supply chain comprised of multiple competing distributors. Such investigations may lead to new policies for improving supply chain efficiency. In addition, we assume that the retail price is fixed. It is common in markets that lowering the retail price results in increased demand. Thus, if the retailer and the distributor cooperate to lower the retail price, both parties can benefit. We are working on an extended version of the model with effort-price dependent demand.
ACKNOWLEDGEMENT

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REFERENCES


