

SIMULATING SCHEDULE OPTIMIZATION PROBLEM IN STEELMAKING CONTINUOUS CASTING PROCESS

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Abstract

This paper establishes the models of the steelmaking continuous casting (SCC) process, and proposed the improved algorithms for this problem. The simulation results of a computerized scheduling system are also given to prove the model. The SCC process scheduling problem is very difficult to get a good performance solution in practice. The scheduling of the SCC process requires that each cast plan is processed on time, and the charges should be processed continuously for the same caster in the same cast, as well as the waiting time of the charges cannot be conflicted mutually in the same converters. We propose a quantum-behaved particle swarm optimization (QPSO) and improved algorithm strategy. The results show that the QPSO is very efficient for solving the SCC production scheduling problem, especially for large scale problem.

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Key Words: SCC, Scheduling Problem, Quantum-Behaved Particle Swarm Optimization

1. INTRODUCTION

Steelmaking Continuous Casting (SCC) production scheduling process is an important operation in iron & steel companies. In this process, we need to decide the sequence, the beginning time, as well as the molten steel machine and scheduling of the molten steel in the iron & steel production stages. It is very difficult to get a good performance solution in practice in iron & steel production [1]. Optimization of the scheduling SCC processes can reduce the energy consumption and improve the profit of the company, so many iron & steel companies have studied the SCC production process for continuous improvement. For example, according to the report of Baoshan Steel Plant in China, one second of SCC production scheduling time saving could bring approximately 5800 dollars profits [2]. However the scheduling of SCC processes has not been solved very well because of its complexity in terms of combinatorial nature, complex constraints, the material continuity and strict practicability requirements, as well as the flow time constrains.

There are many related literature study the optimization for the scheduling of the SCC problems, and many models have been proposed and discussed. However, it is still not very satisfactory when solving the large-scale SCC problems in actual iron & steel companies. The most commonly used methods for solving the SCC scheduling problems are and mixed-integer linear optimization (MILP) [3-5], artificial intelligence, linear optimization (LP), heuristics [6, 7] and simulation methods. The literatures [8-10] illustrate the detailed reviews on these methods.

The two-stage heuristic method is commonly used to solve the machine assignments according to the load equalization and secondly optimized the charges' beginning time. Recently, Li et al. [11] propose a unit-specific event for SSC production scheduling process, and Lin et al. [12] propose a continuous optimal approach for the medium size SSC problem. Then Janak et al. [13, 14] extend the approach to large scale industrial batch plant. Comparing

with the models not use the rolling horizon approach, the final results of the models using the extended rolling horizon approach imply that this improved method can result in the shorter total computing time and better performances of the scheduling solutions. Chen et al. [15] improve the Lagrange Relaxation (LR) approach for the manufacturing job shop scheduling problems, and Fisher [16] employs this method to solve the integer programming production problems. Recently Tang et al. [17] establish a mathematical program model embedded within the LR for SSC production scheduling problem. Almost at the same time, the subgradient method and the bundle method are proposed for job shop scheduling problems [18, 19].

From the observation, however, all the algorithms mentioned above cannot solve the actual SCC process scheduling problems. Form all the previous methods, even employing highly complex algorithm and modern super computers, the compute time used to solve the problems of integer programming are multiplying with increase of the problem size, so the optimization of the SCC process scheduling problems are needed according to the actual steelmaking situations, which is the primary motivation of this research.

2. PROBLEM DESCRIPTION

Generally speaking, it includes smelting process (LD), blowing argon (AR), ladle treatment (LF), vacuum (RH), rotary table process (the CW) and continuous casting (CC) in the process of steelmaking-continuous casting. The process routes may be different for different steels. The model in this paper is based on the practical production situation of an iron & steel company in Ma On Shan in China. There are four sets of parallel machines in smelting process, 3 sets of parallel machines in continuous casting process, and two parallel machines for each other process. The parallel machines are suitable for all of the same process. According to the actual production situation, we consider four different process routes: LD – AR – CW – CC, LD – LF – CW – CC, LD – LF – RH – CW – CC, LD – RH – CW – CC. Fig. 1 illustrates the schematic of the steelmaking continuous casting process routes.

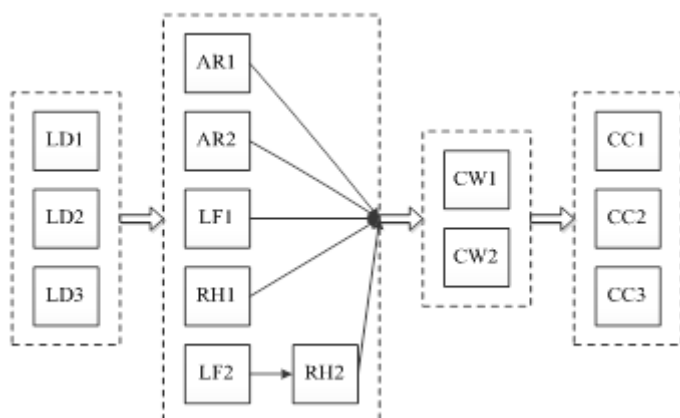


Figure 1: The schematic of the steelmaking continuous casting process routes.

The main purpose of SCC production scheduling is the coordination between smelting, refining, continuous casting production balance problems in order to meet the requirements of the continuous casting molten steel composition and temperature of molten steel, as well as to ensure reasonable cohesion and matching with subsequent rolling process. SCC production scheduling is considered according to the given production plan in this paper, so in this paper, the assumptions are proposed as follows: sequence of charges in casts, sequence of casts and the corresponding continuous casting machine are already known; the beginning time of each cast is known; the assignments of the smelting, refining stage equipment for the castes are not restricted.

3. MATHEMATICAL MODEL

All the symbols used in the model are shown in Table I.

Table I: The symbols of model.

Symbols	Explanation of symbols
N	Total number of the charges
n	Number of the charge, index $n = 1, 2, \dots, N$
M	Total number of the casts
m	Number of the cast, index $m = 1, 2, \dots, M$
k_f	Total number of processes of the m cast
M_n	All the casts of the n charge
i, j, k	The process of the steel
Q_{mi}	The i process of the m cast
r_{mi}	The machine number of the i process of the m cast
s_{mi}	The beginning time of the i process of the m cast
t_{mi}	The finished time of the i process of the m cast
p_{mi}	The process time of the i process of the m cast
d_{mi}	The difference between the s_{mi} and $t_{m(i-1)}$

The mathematical formulations of model are shown as:

$$\min Z = \max_{m=1, \dots, M} \{t_{m k_m}\} \quad (1)$$

$$\text{s. t. } t_{m k_m} = s_{(m+1)k_{m+1}}, \quad m, m+1 = 1, 2, \dots, M_n \quad (2)$$

$$t_{mi} = s_{mi} + p_{mi}, \quad m = 1, 2, \dots, M, \quad 0 < i < k_m \quad (3)$$

$$t_{mi} - s_{m(i+1)} = 0, \quad m = 1, 2, \dots, M, \quad 0 < i < k_m \quad (4)$$

$$s_{mi} - t_{m(i+1)} \leq p_{cw}, \quad p_{cw} = p_5, \quad m = 1, 2, \dots, M \quad (5)$$

$$s_{m5} - t_{m5} \leq 0, \quad p = 1, 2, \dots, v_t, \quad m = 1, 2, \dots, M \quad (6)$$

$$(s_{m_0 i} - s_{m_1 i})(s_{m_0 i} - t_{m_1 i}) \geq 0, \quad m_0 \neq m_1, \quad r_{m_0 i} = r_{m_1 i}, \quad m = 1, 2, \dots, M \quad (7)$$

$$(t_{m_0 i} - s_{m_1 i})(t_{m_0 i} - t_{m_1 i}) \geq 0, \quad m_0 \neq m_1, \quad r_{m_0 i} = r_{m_1 i}, \quad m = 1, 2, \dots, M \quad (8)$$

$$D = \sum_n \sum_m \sum_i (s_{m(i+1)} - t_{mi}), \quad m = 1, 2, \dots, M_n, \quad i = 1, \dots, k_m - 1 \quad (9)$$

Eq. (1) is the objective function, aims to minimize the makespan. Eq. (2) is the constrain of continuous casting. Eq. (3) is the constrain of the process time. Eq. (4) means the constrain of the beginning time and the finish time when all the processes are processed on time. Eqs. (5) and (6) are the time constraints for the buffer processes. Eqs. (8) and (9) mean that it only allows one task for the same time in the same process. Eq. (9) limit the waiting time of the cast. Eq. (9) could be removed if there is no waiting time according to the result of Eq. (4).

4. HYBRID ALGORITHM

In recent years, some researches focused on the optimization of the SCC process scheduling problems based on Particle Swarm Optimization (PSO) algorithm. However, the standard

PSO algorithm is greatly affected by the initial group of particle swarm, and it needs to expand population size and increase the number of iterations to ensure ergodicity of the algorithm in dealing with large-scale problem. To solve the problem, this paper studies the quantum particle swarm optimization (QPSO) algorithm to solve SSC production scheduling problem and put forward improved algorithm strategies.

4.1 Quantum-behaved particle swarm optimization

PSO is a new evolutionary algorithm which has been developed in recent years. Kennedy and Eberhart were inspired from the behaviour of bird populations from predation, and then proposed PSO [20]. In PSO, the solution for each optimization problem is a particle. Each particle has an adaptive value and a speed of its fighting. The positions of particles are identified randomly and the velocity is initialized as 0 in the first iteration. At first the particles update their speed and new position with the formula below:

$$V_{i,j}(t+1) = w \cdot V_{i,j}(t) + c_1 \cdot r_{1,i}(t) \cdot [P_{i,j}(t) - X_{i,j}(t)] + c_2 \cdot r_{2,i}(t) \cdot [P_{g,j}(t) - X_{i,j}(t)] \quad (10)$$

$$X_{i,j}(t+1) = X_{i,j}(t) + V_{i,j}(t+1) \quad (11)$$

Literature on the convergence of the PSO shown that in order to guarantee the convergence of the algorithm, each particle should convergence particle attractor. That is [21]:

$$C_{i,j}(t) = (c_1 \cdot r_1 \cdot P_{i,j}(t) + c_2 \cdot r_2 \cdot P_{g,j}(t)) / (c_1 \cdot r_1 + c_2 \cdot r_2), \quad j = 1, 2, \dots, d, \quad c_1, c_2 \in U(0, 1) \quad (12)$$

$$C_{i,j}(t) = \alpha \cdot P_{i,j}(t) + (1 - \alpha) \cdot P_{g,j}(t), \quad \alpha \in U(0, 1) \quad (13)$$

P_i is the personal best particle and P_g is the global best particle, where $\alpha = (c_1 \cdot r_1) / (c_1 \cdot r_1 + c_2 \cdot r_2)$. It can be seen that the local attractor is a stochastic attractor of particle i that lies in a hyper-rectangle with P_i and P_g being two ends of its diagonal.

On the basis of the study of the convergence process of particles, Sun et al. put forward a quantum-behaved particle swarm optimization algorithm (QPSO) [22]. Because of the removal of the velocity vector, the QPSO algorithm has fewer parameters, and optimizes the search strategy.

According to the literature [23], the quantum state of the quantum state is described by a wave function $\psi(\vec{x}, t)$. Then a global point is introduced in QPSO [24]. Global point $mbest$ is identified as the mean of the $Pbest$ positions of all particles. And the formulation is shown in Eq. (14):

$$mbest(t) = (mbest_1(t), mbest_2(t), \dots, mbest_d(t)) = \left(\frac{1}{n} \sum_{i=1}^n P_{i,1}(t), \frac{1}{n} \sum_{i=1}^n P_{i,2}(t), \dots, \frac{1}{n} \sum_{i=1}^n P_{i,d}(t) \right) \quad (14)$$

Then, the new position of a particle is:

$$X_{i,j}(t+1) = C_{i,j}(t) \pm \beta \cdot |mbest_j - X_{i,j}(t)| \cdot \ln(1/u), \quad u = rand(0, 1) \quad (15)$$

where β is the contraction-expansion coefficient.

Generally speaking, the QPSO is structured in three steps: firstly, the positions of the particles are initialized randomly, and P_i of an initial position of particles is initialized as 0. Secondly, the objective functions, P_i and P_g are identified. Thirdly, the new position and velocity are given.

4.2 Decoding method

The first step for the QPSO algorithm to solve the SCC process is the decoding. For the general SCC process model, the array decoding method can be used. The number of charge can be denoted by the number of array, and the number of cast can be stored in the array. As shown in Fig. 2.

Charge	1	2	...	n	...	N
Cast	1	3	...	k	...	2 1

Figure 2: Array decoding example.

According to the characteristics of the SCC process model in this paper, the array decoding method couldn't cover all processes. So a matrix decoding method is proposed, as shown in Fig. 3.

Charge	1	2	3	4	...	n	...	$N-1$	N
LD	2	1	1	2	3	1 3 2
AR	0	2	0	1	2	0 0 0
LF	2	0	0	0	0	1 1 0
RH	0	0	1	0	0	2 1 2
CW	1	2	1	1	2	1 2 2
CC	2	3	2	3	1	3 1 3

Figure 3: Matrix decoding example.

There is a $6 \times n$ matrix in Fig. 3. And a row of the matrix expresses a process and a column of the matrix express all processes of a charge. While, the first row means LD process, the second row means AR process, the third row means LF process, the fourth row means RH process, the fifth row means CW process, and the last row means CC process. And, the first column means the process routing of charge 1 is: LD2-LF2-CW1-CC2, the second column means the process routing of charge 2 is: LD1-AR2-CW2-CC3, the third column means the process routing of charge 3 is: LD1-RH1-CW1-CC2, and the $N-1$ column means the process routing of charge $N-1$ is: LD3-LF1-RH1-CW2-CC1. In addition, adjust the machine number of each process can get more production path.

5. COMPUTATIONAL EXPERIMENTS

5.1 Parameter selection

The *popsize*, inertia weight parameters, as well as the parameters c_1 and c_2 need to be determined according to the literature [25]. In addition, there is a new parameter contraction-expansion coefficient β in QPSO algorithm. Four functions which are widely used to testify the performance of the algorithm are employed to test the QPSO algorithm under different parameter combinations of properties. This article set the *popsize* to 50. The function expressions are shown in Table II.

Parameter design strategy is as follows:

- a) EX1: $c_1 = c_2 = 2, \omega = [0.4, 0.9], \beta = 0.5$;
- b) EX2: $c_1 = 1.85, c_2 = 2, \omega = [0.4, 0.9], \beta = 1$;
- c) EX3: $c_1 = c_2 = 2, \omega = [0.38, 0.99], \beta = 0.5$;
- d) EX4: $c_1 = 1.85, c_2 = 2, \omega = [0.38, 0.99], \beta = 1$.

50 independent simulation experiments are made for four kinds of parameter strategies respectively, and it stops when the target error arrives.

Table II: Function expressions.

Function	Function expression	Dimension	Value area	Theoretical extreme value	Target error
Sphere	$f(x) = \sum_{i=1}^n x_i^2$	30	$[-100,100]^n$	0	10^{-2}
Rastrigin	$f(x) = \sum_{i=1}^n (x_i^2 - 10\cos(2\pi x_i) + 10)$	30	$[-5.12,5.12]^n$	0	10^{-2}
Griewank	$f(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	$[-600,600]^n$	0	10^{-2}
Rosenbrock	$f(x) = \sum_{i=1}^{n-1} (100(x_{i+1} + x_i^2) + (x_i - 1)^2)$	30	$[-30,30]^n$	0	10^{-2}

The success rate of convergence (*Suc*), the minimum number of iterations (*Min*), the largest number of iterations (*Max*) and the average number of iterations (*Mean*), and calculate the standard deviation (*St. D*) are recorded and the sequence is given according to the *St. D*. The parameter selection experiment results are shown in Table III.

Table III: Parameter selection experiment result.

Function	Type	<i>Suc</i> (%)	<i>Min</i>	<i>Max</i>	<i>Mean</i>	<i>St. D</i>	Sequence
Sphere	EX1	100	582	2440	1394	277.4	4
	EX2	100	321	583	414	81	2
	EX3	100	201	374	264	51.4	1
	EX4	99.7	424	992	718	178.2	3
Rastrigin	EX1	100	990	2776	1912	365	4
	EX2	100	142	649	401	119.3	2
	EX3	99.8	141	571	342	78.5	1
	EX4	100	322	1720	1036	127	3
Griewank	EX1	100	152	787	372	27.8	2
	EX2	100	53	153	119	29	4
	EX3	100	34	112	67	25	1
	EX4	100	102	216	130	28.6	3
Rosenbrock	EX1	100	568	1306	898	139	1
	EX2	100	250	1456	873	333	3
	EX3	100	247	871	672	184.5	2
	EX4	100	753	2166	1573	431	4

Table III shows that the performance of EX3 is the best among all parameter selection experiments, so the parameters of EX3 ($c_1 = c_2 = 2$, $\omega = [0.38, 0.99]$, $\beta = 0.5$) are selected for the further experiment.

5.2 Simulation experiment

The experimental data come from the actual production situation of a large-size iron & steel company in China. Select the order and production data in the second quarter of 2015 (April, May and June) of the factory for experiments. Experiments are carried out in a parallel state for ten times, and calculate the average, as shown in Table IV.

Table IV: Calculation results.

Sample	Calculate times (min)	PSO (10 ³ h)	QPSO (10 ³ h)
April	5	7.99	6.89
May	5	6.85	5.78
June	5	8.03	6.54
Second quarter	10	25.41	20.15

From the data of Table IV, compared with the traditional PSO algorithm, the QPSO that proposed in this paper can get better results. Therefore, the QPSO algorithm has better performance in the process of dealing with large-scale SCC process problem.

6. CONCLUSIONS

Optimization of the scheduling SCC processes can reduce the energy consumption and the productions cost, as well as increase the profit, so many iron & steel companies have studied the SCC production process for continuous improvement. We study the optimal models for the SCC problem. It is very difficult to get a good performance solution in practice in iron & steel production, especially for the large-scale problems. We employ QPSO algorithm and put forward the improved algorithm strategies to solve SSC production scheduling problem. It has been proved that comparing with the PSO algorithm, the QPSO algorithm is a very efficient approach and it can help the companies get the scheduling results in the shorter total computing time and better performances for the SCC process scheduling problem, especially when dealing with large scale problem. In addition, the QPSO algorithm can escape effectively from local optimum compared with PSO algorithm. The simulation results also confirm the conclusions.

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