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COOPERATIVE INVENTORY STRATEGY IN A DUAL-CHANNEL SUPPLY CHAIN WITH TRANSSHIPMENT CONSIDERATION

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Abstract
Aimed at the possibility of stock-out of the traditional retail channel under stochastic demand in a dual-channel supply chain, we propose a cooperative inventory strategy, in which the excess demand of the traditional retail channel is complemented by the excess inventory of the manufacturer's direct channel through transshipment. We establish a newsboy model to analyse each supply chain member's order quantity decision making process, and prove the unique existence of pure-strategy Nash equilibrium under the uncooperative inventory strategy and the cooperative inventory strategy. We go on to discuss the impacts of the wholesale price, the channel substitution rate and the transshipment cost on optimal order quantities under each inventory strategy. Numerical simulation shows that when values of the wholesale price and the transshipment price satisfy certain conditions, the cooperative inventory strategy can not only lead to an increased supply chain profit but also achieve Pareto improvements of both the manufacturer and the retailer.
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Key Words: Supply Chain, Dual Channel, Inventory Cooperation, Transshipment, Game Theory

1. INTRODUCTION

When the order quantity cannot satisfy the demand in a retail channel, the stock-out arises and the retailer loses profit if no remedial measure is taken. To reduce the loss of profits when stock-out arises, some retailers apply for the replenishment from manufacturers, while other retailers apply for the transshipment from other retail channels. However, the replenishment of products having a short life cycle from manufacturers is costly because of the long lead time (e.g., productive material preparation, production and transportation) and the short sales cycle. As a result, the transshipment of products having a short life cycle between retail channels is preferred by retailers. For example, some famous fashion firms, such as ZARA and H&M, adopt transshipments to solve the problem of stock-out [1, 2].

With the high-speed development of e-commerce and increased consumer acceptance of online shopping, more and more manufacturers, including ZARA, HP, Apple and Lenovo, adopt a dual-channel supply chain to sell their products. In a dual-channel supply chain, the transshipment from the manufacturer-controlled direct channel to the retailer-controlled channel provides a method for solving the problem of stock-out in the traditional retail channel. Both channels share the benefit of the transshipment policy through the transshipment price, but the transshipment policy is not without its disadvantages. If the transshipment policy is adopted, the retailer places a smaller order quantity and may realize a decrease in retail profit, and the manufacturer also realizes a decrease in wholesale profit in the traditional retail channel and the increase of inventory surplus in the direct channel. Therefore, both supply chain members should seriously consider whether or not to adopt the transshipment policy.

The transshipment problem in a dual-channel supply chain has received little research attention. Based on this, we consider transshipment as a cooperative inventory strategy between the direct channel and the traditional retail channel. We employ a newsboy model to analyse the decision making process, investigate the existence of unique Nash equilibrium of order quantities, and simulate the variation of optimal order quantities and profits when the wholesale price, the transshipment cost and the transshipment price change. Furthermore, we examine conditions that the cooperative inventory strategy can outperform the uncooperative inventory strategy. It is unreasonable to tranship the product from the retailer back to the manufacturer. Therefore, we do not consider the transshipment from the traditional retail channel to the direct channel controlled by the manufacturer when the direct channel experiences stock-out. This case can, however, be easily presented in our model.

Based on the above analyses, we come to the following results: (1) when the wholesale price and the transshipment price satisfy certain conditions, the cooperative inventory strategy is better than the uncooperative inventory strategy from the perspective of individual members; and (2) not only the wholesale price and the transshipment price but also the transshipment cost and each channel's substitution rate are important in the decision making of each supply chain member.

This paper is organized as follows. In Section 2, we review relevant research. In Section 3, we describe the problem and formulate the model. In Section 4, we analyse the model and provide the theoretical results. In Section 5, we present some numerical examples. In Section 6, we summarize the paper.

2. LITERATURE REVIEW

This paper is related to the literature of transshipment and dual-channel supply chain. As for the transshipment, some researchers have studied the transshipment between traditional retailers without considering the role of the manufacturer, while other researchers have focused on the two-echelon supply chain with the manufacturer being involved. As for the dual-channel supply chain, the inventory competition, coordination and cooperation under stochastic demand attract considerable attention of researchers.

The transshipment problem in the one-echelon supply chain has been well studied. For example, Rudi et al. [3] studied a one-period transshipment problem between traditional retail channels, and derived the transshipment price that induced retailers to take order quantities of traditional retail channels that maximized the profit of the entire supply chain. In addition to the linear transshipment price, Hu et al. [4] argued that other contracts were necessary to coordinate the transshipment between traditional retail channels. Based on the conclusion in Hu et al. [4], Hezarkhani and Kubiak [5] used the Generalized Nash Bargaining Solution to design a contract. The contract could not only induce retailers to choose order quantities of traditional retail channels which maximized the profit of the entire supply chain, but also divides extra profits created by the transshipment based on the bargaining power of traditional retail channels. Ben et al. [6] considered the transshipment issue with fuzzy demand and service level constraints. Tiacci and Saetta [7] considered the transshipment strategy based on inventory availability and the transshipment strategy based on inventory equalization, and found that both transshipment strategies were more effective than the non-lateral transshipment strategy under certain conditions. Van der Heide and Roodbergen [8] studied optimal transshipment and rebalancing strategies for library books. Glazebrook et al. [9] proposed a hybrid transshipment strategy which exploited economies of scale by moving excess inventory between traditional retail channels to prevent future stock-out in addition to meeting immediate stock-out. Çömez-Dolgan and Fescioglu-Unver [10] studied the question of whether or not a retailer should always satisfy the transshipment request from other retailers, and suggested that it was centrally optimal to satisfy all the transshipment requests when only few retailers stayed in the system. Çömez et al. [11] built a flexible transshipment policy that allowed the retailer to reject a transshipment request on one day but accepted another transshipment request a few days later, instead of extreme transshipment strategies such as complete inventory sharing or no inventory sharing. Liang et al. [12] studied the optimal transshipment strategy for a firm which owned two retail stores, and analysed the impacts of transshipment and demand distribution shapes on the optimal transshipment strategy. Paterson et al. [13] reviewed the research on transshipment in the one-echelon supply chain.

The literature of the transshipment in the one-echelon supply chain does not consider the effect of the manufacturer's wholesale price on the order competition and coordination between traditional retail channels. In practice, however, the manufacturer's wholesale price decision plays a substantial role in the traditional supply chain. Thus, some research projects began to focus on the transshipment in the two-echelon supply chain. Dong and Rudi [14] studied how the manufacturer and the retailer benefited from the transshipment in a two-echelon supply chain under an exogenous and an endogenous wholesale price respectively. Zhang [15] extended some of the main results in [14] to general demand distributions. Archibald et al. [16] deployed an approximate stochastic dynamic programming approach to determine whether a retailer's stock-out should be complemented by an emergency supply from the manufacturers or by the transshipment from other retailers. Shao et al. [17] suggested that a low transshipment price could mitigate the problem of overstocked inventories, but would decrease the manufacturer's wholesale profit. Dong et al. [18] showed that when the cost of transshipment was incurred by one retailer while the benefit of transshipment was enjoyed by other retailers, the transshipment strategy was difficult to implement. Therefore, they devised a transshipment contract for a manufacturer to encourage adopting the transshipment between traditional retail channels.

This paper is also related to the inventory competition in a dual-channel supply chain under stochastic demand. Boyaci [19] showed that when stock-out occurred either in the traditional retail channel or in the direct channel, the consumer switching behaviour resulted in the inventory competition between two channels. Geng and Mallik [20] proved that the inventory competition might not only result in overstocked inventories in either channel but also induce a manufacturer to cut a retailer's order even when the capacity was unlimited. To improve the efficiency of the dual-channel supply chain, some researchers designed various contracts to coordinate the inventory decision making of individual members [21], while other researchers tried to explore the inventory cooperation between two channels. For example, Gallino and Moreno [22] implemented a "buy-online, pick-up-in-store" project which needed two channels to share reliable inventory availability information. Zhang [23] proposed the online pre-sales in an apparel dual-channel supply chain, and showed that the online pre-sales provided guidance for the traditional retail channel, and suggested that the online pre-sales helped to resolve the problem of overstocked inventories. However, the transshipment between the traditional retail channel and the direct channel which is an important form of the inventory cooperation has not been studied before.

3. PROBLEM DESCRIPTION AND MODEL

Considering a dual-channel supply chain in a selling season, the supply chain consists of a manufacturer and a traditional retailer. Each supply chain member is risk-neutral and completely rational. The manufacturer, the retailer and the supply chain are represented by the subscript "*m*", subscript "*r*" and subscript "*c*" respectively. The retailer purchases the product at a unit wholesale price w from the manufacturer and sells to consumers at a unit retail price p_r. The manufacturer directly sells to consumers through its own direct channel at a unit direct sale price p_m as well. c_r and c_m which are incurred by the manufacturer are unit selling costs of the traditional retail channel and the direct channel. s_r and s_m are the unit salvage values for the leftover inventory of the traditional retail channel and the direct channel. For simplicity, no unit cost for production or no penalty cost for lost sales is assumed. All price parameters are exogenous in the paper. To avoid trivial cases, we assume $p_r > w > c_r > s_r$ and $p_m > c_m > s_m$.

Demands of the traditional retail channel and the direct channel, $\tilde{D}_r$ and $\tilde{D}_m$ are random and independent of each other. Let $f(x)$ and $g(y)$ represent the density functions for $\tilde{D}_r$ and $\tilde{D}_m$. $F(x)$ and $G(y)$ represent the cumulative density functions for $\tilde{D}_r$ and $\tilde{D}_m$. Order quantities of the traditional retail channel and the direct channel, Q_r and Q_m, are placed before the selling season starts. After orders are placed, the product is immediately delivered, and then demands are realized.

Two inventory strategies are considered: the uncooperative inventory strategy and the cooperative inventory strategy. The uncooperative inventory strategy is represented by the superscript D. If the uncooperative inventory strategy is adopted, some unsatisfied consumers in one channel will switch to another channel [19-21]. We let k_r represent the ratio of the excess demand of the traditional retail channel which switches to the direct channel; i.e., the substitution rate from the traditional retail channel. The direct channel obtains the additional revenue $p_m E\min((Q_m - \tilde{D}_m)^+, k_r(\tilde{D}_r - Q_r)^+)$ alone if the products are in inventory. It is worth noting that $x^+ = \max(x, 0)$. Similarly, we let k_m represent the substitution rate from the direct channel. The traditional retail channel obtains the additional revenue $p_r E\min((Q_r - \tilde{D}_r)^+, k_m(\tilde{D}_m - Q_m)^+)$ alone if the products are in inventory. Expected profits of the supply chain members under the uncooperative inventory strategy are:

$$\begin{aligned}\pi_m^D &= p_m E\min\left(\tilde{D}_m, Q_m\right) + p_m E\min\left(\left(Q_m - \tilde{D}_m\right)^+, k_r\left(\tilde{D}_r - Q_r\right)^+\right) \\ &\quad + s_m E\left(Q_m - \tilde{D}_m - k_r\left(\tilde{D}_r - Q_r\right)^+\right)^+ + wQ_r - c_r Q_r - c_m Q_m\end{aligned} \tag{1}$$

$$\begin{aligned}\pi_r^D &= p_r E\min\left(\tilde{D}_r, Q_r\right) + p_r E\min\left(\left(Q_r - \tilde{D}_r\right)^+, k_m\left(\tilde{D}_m - Q_m\right)^+\right) \\ &\quad + s_r E\left(Q_r - \tilde{D}_r - k_m\left(\tilde{D}_m - Q_m\right)^+\right)^+ - wQ_r\end{aligned} \tag{2}$$

The cooperative inventory strategy is represented by the superscript N. If the cooperative inventory strategy is adopted, the excess demand of the traditional retail channel is complemented by the excess inventory of the direct channel through transshipment. The revenue of transshipment is allocated by the transshipment price p_t. The direct channel pays the unit transshipment cost c_t. The transshipment is immediately completed. To guarantee that the transshipment occurs if and only if the traditional retail channel is out of stock and the direct channel has excess inventory, we assume that $w < c_m + c_t$, $s_r < s_m + c_t$, $p_r < p_m + c_t$, $s_m + c_t \le p_t$ and $p_t \le p_r$. Expected profits of supply chain members under the cooperative inventory strategy are:

$$\begin{aligned}\pi_m^N &= p_m E\min\left(\tilde{D}_m, Q_m\right) + \left(p_t - c_t\right) E\min\left(\left(Q_m - \tilde{D}_m\right)^+, \left(\tilde{D}_r - Q_r\right)^+\right) \\ &\quad + s_m E\left(Q_m - \tilde{D}_m - \left(\tilde{D}_r - Q_r\right)^+\right)^+ + wQ_r - c_r Q_r - c_m Q_m\end{aligned} \tag{3}$$

$$\begin{aligned}\pi_r^N &= p_r E\min\left(\tilde{D}_r, Q_r\right) + p_r E\min\left(\left(Q_r - \tilde{D}_r\right)^+, k_m\left(\tilde{D}_m - Q_m\right)^+\right) - wQ_r \\ &\quad + \left(p_r - p_t\right) E\min\left(\left(Q_m - \tilde{D}_m\right)^+, \left(\tilde{D}_r - Q_r\right)^+\right) + s_r E\left(Q_r - \tilde{D}_r - k_m\left(\tilde{D}_m - Q_m\right)^+\right)^+\end{aligned} \tag{4}$$

4. MODEL AND ANALYSIS

4.1 Case 1: the uncooperative inventory strategy

We first examine the optimal order quantity decisions of the traditional retail channel and the direct channel under the uncooperative inventory strategy.

Taking the first derivative of π_r^D and π_m^D with respect to Q_r and Q_m, we get:

$$\frac{\partial \pi_r^D}{\partial Q_r} = p_r - w - \left(p_r - s_r\right)\left(\int_0^{Q_r}\int_0^{Q_m} f\left(x\right)g\left(y\right)dydx + \int_{Q_m}^{Q_m+\frac{Q_r}{k_m}}\int_0^{Q_r-k_m(y-Q_m)} f\left(x\right)g\left(y\right)dydx\right) \tag{5}$$

$$\frac{\partial \pi_m^D}{\partial Q_m} = p_m - c_m - \left(p_m - s_m\right)\left(\int_0^{Q_r}\int_0^{Q_m} f\left(x\right)g\left(y\right)dydx + \int_{Q_r}^{Q_r+\frac{Q_m}{k_r}}\int_0^{Q_m-k_r(x-Q_r)} f\left(x\right)g\left(y\right)dydx\right) \tag{6}$$

By letting $\frac{\partial \pi_r^D}{\partial Q_r}$ and $\frac{\partial \pi_m^D}{\partial Q_m}$ be zero，we can obtain the unique Nash equilibrium of Q_r^D and Q_m^D. Next, we use Proposition 1 to prove the existence of the unique Nash equilibrium.

Proposition 1. The unique Nash equilibrium of Q_r^D and Q_m^D exists under the uncooperative inventory strategy.

Proof:

$$\frac{\partial^2 \pi_r^D}{\partial Q_r^2} = -\left(p_r - s_r\right)\left(\int_0^{Q_m} f\left(Q_r\right)g\left(y\right)dy + \int_{Q_m}^{Q_m+\frac{Q_r}{k_m}} f\left(Q_r - k_m(y-Q_m)\right)g\left(y\right)dy\right) < 0 \tag{7}$$

$$\frac{\partial^2 \pi_m^D}{\partial Q_m^2} = -\left(p_m - s_m\right)\left(\int_0^{Q_r} f\left(x\right)g\left(Q_m\right)dx + \int_{Q_r}^{Q_r+\frac{Q_m}{k_r}} f\left(x\right)g\left(Q_m - k_r(x-Q_r)\right)dx\right) < 0 \tag{8}$$

$$\frac{\partial^2 \pi_r^D}{\partial Q_r \partial Q_m} = -k_m\left(p_r - s_r\right)\int_{Q_m}^{Q_m+\frac{Q_r}{k_m}} f\left(Q_r - k_m(y-Q_m)\right)g\left(y\right)dy < 0 \tag{9}$$

$$\frac{\partial^2 \pi_m^D}{\partial Q_m \partial Q_r} = -k_r\left(p_m - s_m\right)\int_{Q_r}^{Q_r+\frac{Q_m}{k_r}} f\left(x\right)g\left(Q_m - k_r(x-Q_r)\right)dx < 0 \tag{10}$$

From Eqs. (7) and (9), we get: $\left|\frac{\partial^2 \pi_r^D}{\partial Q_r^2}\right| > \left|\frac{\partial^2 \pi_r^D}{\partial Q_r \partial Q_m}\right|$. From Eqs. (8) and (10), we get: $\left|\frac{\partial^2 \pi_m^D}{\partial Q_m^2}\right| > \left|\frac{\partial^2 \pi_m^D}{\partial Q_m \partial Q_r}\right|$. Furthermore, $H = \begin{vmatrix} \frac{\partial^2 \pi_r^D}{\partial Q_r^2} & \frac{\partial^2 \pi_r^D}{\partial Q_r \partial Q_m} \\ \frac{\partial^2 \pi_m^D}{\partial Q_m \partial Q_r} & \frac{\partial^2 \pi_m^D}{\partial Q_m^2} \end{vmatrix} > 0$. Therefore, profits of the supply chain members are strictly concave in their own order quantities under the uncooperative inventory strategy.

Proposition 2. When the wholesale price w becomes larger, the order quantity Q_r^D becomes smaller, while the order quantity Q_m^D becomes larger.

Proof: It is easy to obtain $\frac{\partial^2 \pi_r^D}{\partial Q_r \partial w} = -1$, and $\frac{\partial^2 \pi_m^D}{\partial Q_m \partial w} = 0$ from Eqs. (5) and (6). Furthermore, according to the implicit function theorem, we obtain:

$$\frac{dQ_r^D}{dw}=-\frac{1}{H}\cdot\begin{vmatrix}\frac{\partial^2\pi_r^D}{\partial Q_r\partial w} & \frac{\partial^2\pi_r^D}{\partial Q_r\partial Q_m}\\ \frac{\partial^2\pi_m^D}{\partial Q_m\partial w} & \frac{\partial^2\pi_m^D}{\partial Q_m^2}\end{vmatrix}<0 \text{ and } \frac{dQ_m^D}{dw}=-\frac{1}{H}\cdot\begin{vmatrix}\frac{\partial^2\pi_r^D}{\partial Q_r^2} & \frac{\partial^2\pi_r^D}{\partial Q_r\partial w}\\ \frac{\partial^2\pi_m^D}{\partial Q_m\partial Q_r} & \frac{\partial^2\pi_m^D}{\partial Q_m\partial w}\end{vmatrix}>0 .$$

Both the double marginalization problem and the horizontal channel competition exist in the dual-channel supply chain. Proposition 2 is exactly in line with the double marginalization problem, thus, the increasing wholesale price decreases the order quantity of the traditional channel Q_r^D. Also, the expected decreasing order quantity Q_r^D provides a larger probability of being out of stock in the traditional retail channel. Due to the horizontal competition consideration, the manufacturer orders a larger quantity of the direct channel Q_m^D so as to satisfy the switched consumers from the traditional channel.

Proposition 3. When the substitution rate k_r becomes larger, the order quantity Q_r^D becomes smaller, while the order quantity Q_m^D becomes larger. When the substitution rate k_m becomes larger, the order quantity Q_r^D becomes larger, while the order quantity Q_m^D becomes smaller.

Proof: From Eqs. (5) and (6), we get: $\frac{\partial^2\pi_r^D}{\partial Q_r\partial k_r}=0$, and $\frac{\partial^2\pi_m^D}{\partial Q_m\partial k_r}>0$. Furthermore, according to the implicit function theorem, we obtain:

$$\frac{dQ_r^D}{dk_r}=-\frac{1}{H}\cdot\begin{vmatrix}\frac{\partial^2\pi_r^D}{\partial Q_r\partial k_r} & \frac{\partial^2\pi_r^D}{\partial Q_r\partial Q_m}\\ \frac{\partial^2\pi_m^D}{\partial Q_m\partial k_r} & \frac{\partial^2\pi_m^D}{\partial Q_m^2}\end{vmatrix}<0 \text{, and } \frac{dQ_m^D}{dk_r}=-\frac{1}{H}\cdot\begin{vmatrix}\frac{\partial^2\pi_r^D}{\partial Q_r^2} & \frac{\partial^2\pi_r^D}{\partial Q_r\partial k_r}\\ \frac{\partial^2\pi_m^D}{\partial Q_m\partial Q_r} & \frac{\partial^2\pi_m^D}{\partial Q_m\partial k_r}\end{vmatrix}>0 .$$

Similarly, we can proof that $\frac{dQ_r^D}{dk_m}>0$ and $\frac{dQ_m^D}{dk_m}<0$.

Proposition 3 indicates that the manufacturer orders a larger quantity Q_m^D when more unsatisfied consumers of the traditional retail channel prefer to switch to the direct channel. Forecasting the manufacturer's choice, the retailer orders a smaller quantity Q_r^D. Similarly, the retailer orders a larger quantity Q_r^D when the unsatisfied consumers of the direct channel become more willing to search for the product in the traditional retail channel. Forecasting the retailer's choice, the manufacturer orders a smaller quantity Q_m^D.

4.2 Case 2: the cooperative inventory strategy

We now consider the optimal order quantity decisions of the traditional retail channel and the direct channel under the cooperative inventory strategy.

Taking the first derivative of π_r^N and π_m^N with respect to Q_r and Q_m we get:

$$\begin{aligned}\frac{\partial\pi_r^N}{\partial Q_r}&=p_r-w-(p_r-p_t)\int_0^{Q_m}\int_{Q_r}^{Q_r+Q_m-y}f(x)g(y)dydx\\&\quad-(p_r-s_r)\left(\int_{Q_m}^{Q_m+\frac{Q_r}{k_m}}\int_0^{Q_r-k_m(y-Q_m)}f(x)g(y)dydx+\int_0^{Q_r}\int_0^{Q_m}f(x)g(y)dydx\right)\end{aligned}\tag{11}$$

$$\begin{aligned}\frac{\partial\pi_m^N}{\partial Q_m}&=p_m-c_m-(p_m-p_t+c_t)\int_{Q_r}^{+\infty}\int_0^{Q_m}f(x)g(y)dydx\\&\quad-(p_t-c_t-s_m)\int_0^{Q_m}\int_{Q_r}^{Q_r+Q_m-y}f(x)g(y)dydx-(p_m-s_m)\int_0^{Q_r}\int_0^{Q_m}f(x)g(y)dydx\end{aligned}\tag{12}$$

By setting $\frac{\partial \pi_r^N}{\partial Q_r}$ and $\frac{\partial \pi_m^N}{\partial Q_m}$ as zero, we can obtain the unique Nash equilibrium of Q_r^N and Q_m^N. Next, we use Proposition 4 to prove the existence of the unique Nash equilibrium.

Proposition 4. The unique Nash equilibrium of Q_r^N and Q_m^N exists under the cooperative inventory strategy.

Proof:

$$\begin{aligned}\frac{\partial^2 \pi_r^N}{\partial Q_r^2} = &-\left(p_t - s_r\right)\left(\int_0^{Q_m} f\left(Q_r\right) g\left(y\right) dy + \int_{Q_m}^{Q_m+\frac{Q_r}{k_m}} f\left(Q_r - k_m(y-Q_m)\right) g\left(y\right) dy\right) \\ &-\left(p_r - p_t\right)\int_0^{Q_m} f\left(Q_r + Q_m - y\right) g\left(y\right) dy < 0\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial^2 \pi_m^N}{\partial Q_m^2} = &-\left(p_m - p_t + c_t\right)\int_{Q_r}^{+\infty} f\left(x\right) g\left(Q_m\right) dx - \left(p_t - s_m\right)\int_0^{Q_r} f\left(x\right) g\left(Q_m\right) dx \\ &-\left(p_t - c_t - s_m\right)\int_0^{Q_m} f\left(Q_r + Q_m - y\right) g\left(y\right) dy < 0\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial^2 \pi_r^N}{\partial Q_r \partial Q_m} = &-\left(p_r - p_t\right)\int_0^{Q_m} f\left(Q_r + Q_m - y\right) g\left(y\right) dy \\ &-k_m\left(p_r - s_r\right)\int_{Q_m}^{Q_m+\frac{Q_r}{k_m}} f\left(Q_r - k_m(y-Q_m)\right) g\left(y\right) dy < 0\end{aligned} \tag{15}$$

$$\frac{\partial^2 \pi_m^N}{\partial Q_m \partial Q_r} = -\left(p_t - c_t - s_m\right)\int_0^{Q_m} f\left(Q_r + Q_m - y\right) g\left(y\right) dy < 0 \tag{16}$$

From Eqs. (13) and (15), we get: $\left|\frac{\partial^2 \pi_r^N}{\partial Q_r^2}\right| > \left|\frac{\partial^2 \pi_r^N}{\partial Q_r \partial Q_m}\right|$. From Eqs. (14) and (16), we get: $\left|\frac{\partial^2 \pi_m^N}{\partial Q_m^2}\right| > \left|\frac{\partial^2 \pi_m^N}{\partial Q_m \partial Q_r}\right|$. Furthermore, $H = \begin{vmatrix} \frac{\partial^2 \pi_r^N}{\partial Q_r^2} & \frac{\partial^2 \pi_r^N}{\partial Q_r \partial Q_m} \\ \frac{\partial^2 \pi_m^N}{\partial Q_m \partial Q_r} & \frac{\partial^2 \pi_m^N}{\partial Q_m^2} \end{vmatrix} > 0$. Therefore, profits of the supply chain members are strictly concave in their own order quantities under the cooperative inventory strategy.

Proposition 5. When the wholesale price w becomes larger, the order quantity Q_r^N becomes smaller, while the order quantity Q_m^N becomes larger.

Proof: It is easy to obtain: $\frac{\partial^2 \pi_r^N}{\partial Q_r \partial w} = -1$, and $\frac{\partial^2 \pi_m^N}{\partial Q_m \partial w} = 0$ from Eqs. (11) and (12). Furthermore, according to the implicit function theorem, we obtain:

$$\frac{dQ_r^N}{dw} = -\frac{1}{H}\cdot\begin{vmatrix} \frac{\partial^2 \pi_r^N}{\partial Q_r \partial w} & \frac{\partial^2 \pi_r^N}{\partial Q_r \partial Q_m} \\ \frac{\partial^2 \pi_m^N}{\partial Q_m \partial w} & \frac{\partial^2 \pi_m^N}{\partial Q_m^2} \end{vmatrix} < 0 \text{ and } \frac{dQ_m^N}{dc_t} = -\frac{1}{H}\cdot\begin{vmatrix} \frac{\partial^2 \pi_r^N}{\partial Q_r^2} & \frac{\partial^2 \pi_r^N}{\partial Q_r \partial w} \\ \frac{\partial^2 \pi_m^N}{\partial Q_m \partial Q_r} & \frac{\partial^2 \pi_m^N}{\partial Q_m \partial w} \end{vmatrix} > 0.$$

Proposition 5 indicates that the double marginalization problem is still present under the cooperative inventory strategy. However, when the wholesale price w decreases, the order quantity of the traditional retail channel Q_r^N closes to the one in the centralized system. Forecasting a larger order quantity of the traditional retail channel Q_r^N, the manufacturer orders a smaller quantity of the direct channel Q_m^N.

Proposition 6. When the transshipment cost c_t becomes larger, the order quantity Q_r^D becomes larger, while the order quantity Q_m^D becomes smaller.

Proof: From Eqs. (11) and (12), we can obtain: $\frac{\partial^2 \pi_r^N}{\partial Q_r \partial c_t} = 0$ and $\frac{\partial^2 \pi_m^N}{\partial Q_m \partial c_t} < 0$. Furthermore, according to the implicit function theorem, we obtain:

$$\frac{dQ_r^N}{dc_t} = -\frac{1}{H} \cdot \begin{vmatrix} \frac{\partial^2 \pi_r^N}{\partial Q_r \partial c_t} & \frac{\partial^2 \pi_r^N}{\partial Q_r \partial Q_m} \\ \frac{\partial^2 \pi_m^N}{\partial Q_m \partial c_t} & \frac{\partial^2 \pi_m^N}{\partial Q_m^2} \end{vmatrix} > 0 \text{ and } \frac{dQ_m^N}{dc_t} = -\frac{1}{H} \cdot \begin{vmatrix} \frac{\partial^2 \pi_r^N}{\partial Q_r^2} & \frac{\partial^2 \pi_r^N}{\partial Q_r \partial c_t} \\ \frac{\partial^2 \pi_m^N}{\partial Q_m \partial Q_r} & \frac{\partial^2 \pi_m^N}{\partial Q_m \partial c_t} \end{vmatrix} < 0.$$

Proposition 6 indicates that when the transshipment cost c_t increases, the manufacturer obtains a lower profit under the cooperative inventory strategy, which induces the manufacturer to order a smaller quantity of the direct channel Q_m^D. Forecasting this, the retailer orders a larger quantity of the traditional retail channel Q_r^D.

Proposition 7. At least one channel orders a smaller quantity under the cooperative inventory strategy when compared with the uncooperative inventory strategy.

Proof: By setting $\frac{\partial \pi_r^N}{\partial Q_r^N}$ and $\frac{\partial \pi_r^D}{\partial Q_r^D}$ as zero, we can obtain:

$$\begin{aligned} & p_r - w - \left(p_r - s_r\right)\left(\int_0^{Q_r^N}\int_0^{Q_m^N} f(x)g(y)\,dydx + \int_{Q_m^N}^{Q_m^N+\frac{Q_r^N}{k_m}}\int_0^{Q_r^N - k_m(y-Q_m^N)} f(x)g(y)\,dydx\right) \\ & = \left(p_r - p_t\right)\int_0^{Q_m^N}\int_{Q_r^N}^{Q_r^N+Q_m^N-y} f(x)g(y)\,dydx \end{aligned} \tag{17}$$

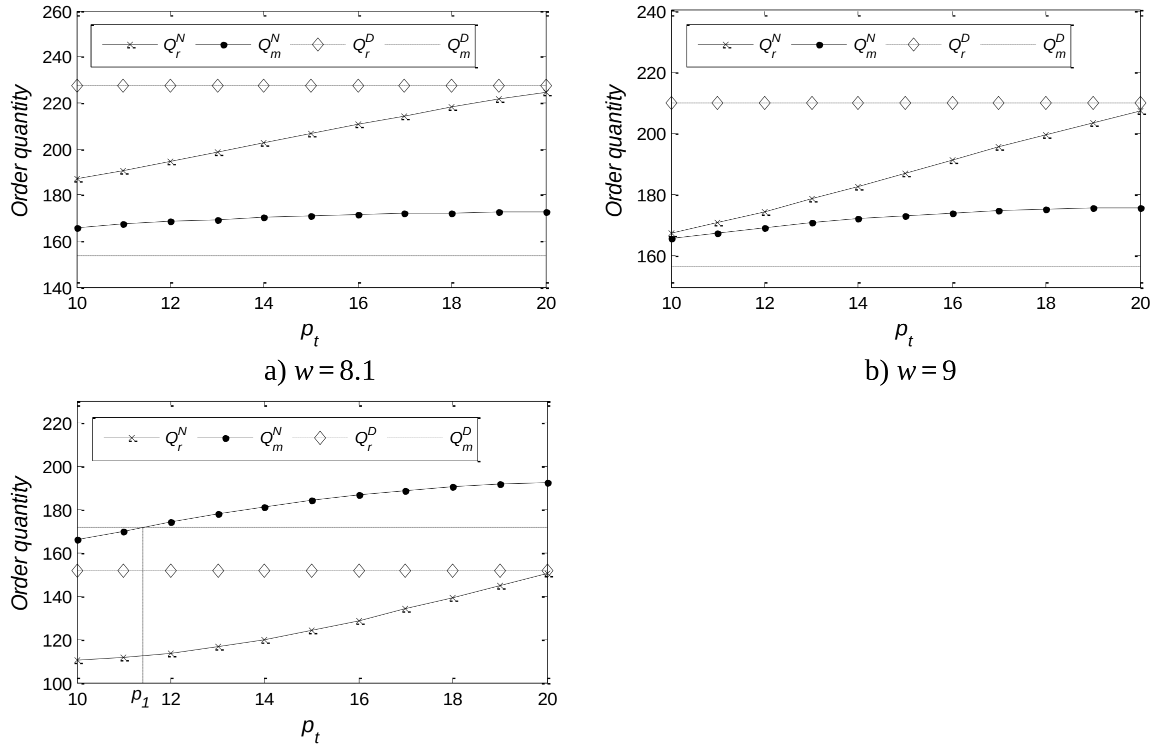
$$p_r - w - \left(p_r - s_r\right)\left(\int_0^{Q_r^D}\int_0^{Q_m^D} f(x)g(y)\,dydx + \int_{Q_m^D}^{Q_m^D+\frac{Q_r^D}{k_m}}\int_0^{Q_r^D - k_m(y-Q_m^D)} f(x)g(y)\,dydx\right) = 0 \tag{18}$$

The left part of Eq. (18) is larger than the left part of Eq. (17) if $Q_r^D < Q_r^N$ and $Q_m^D < Q_m^N$. However, the right part of Eq. (18) is smaller than the right part of Eq. (17) since $p_t \le p_r$. Obviously, Eqs. (17) and (18) cannot establish simultaneously if $Q_r^D < Q_r^N$ and $Q_m^D < Q_m^N$. Therefore, Proposition 7 can be proved by the contradiction.

5. NUMERICAL EXAMPLES

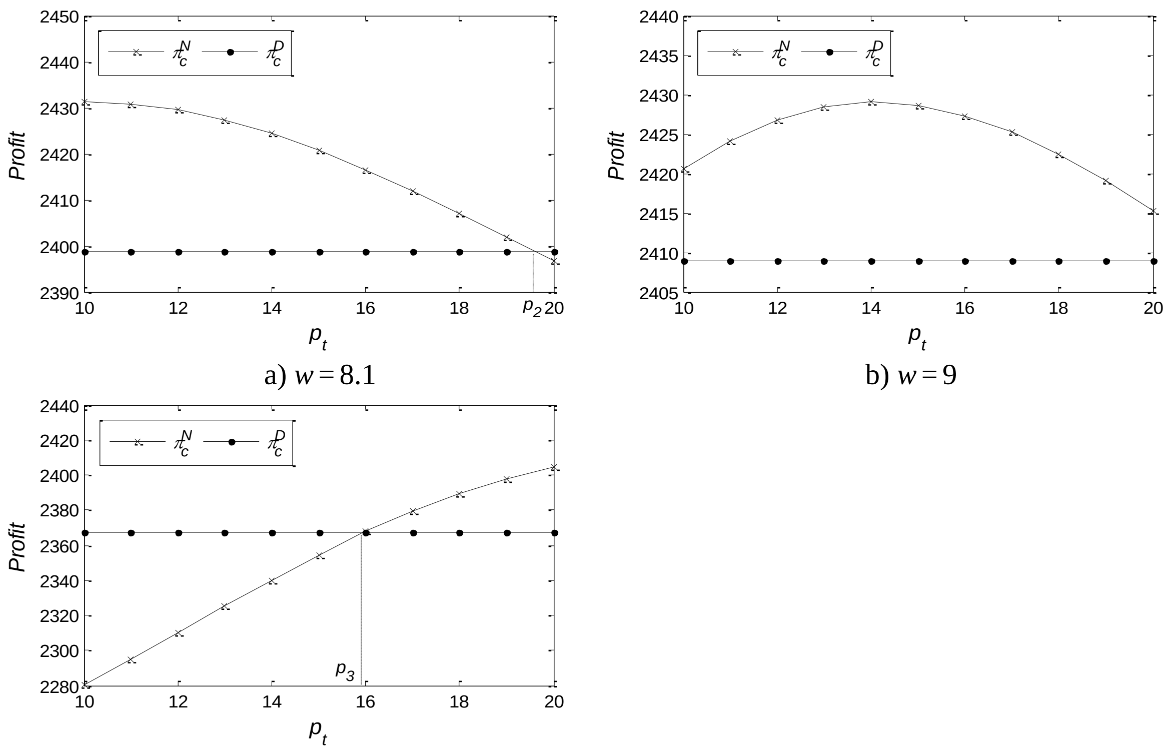
To conduct more sensitive analyses and investigate the condition that the cooperative inventory strategy is adopted, we present following numerical examples. We specify these parameters as follows: $p_r = 20$, $p_m = 18$, $c_r = 8$, $c_m = 6.4$, $s_r = s_m = 4$, $k_m = 0.8$, $k_r = 0.5$, $c_t = 6$, $\tilde{D}_r \sim U[0, 300]$ and $\tilde{D}_m \sim U[0, 200]$.

The transshipment provides a second chance for the traditional retail channel to satisfy the demand. Thus, Fig. 1 shows that the traditional retailer always places a smaller order quantity if the cooperative inventory strategy is adopted. We may assume that the chance to get extra profit from the transshipment always induces the manufacturer to order a larger quantity of the direct channel. However, the total profit of the manufacturer consists of the wholesale profit and direct sale profit and the transshipment profit under the cooperative inventory strategy. Fig. 1 shows that when the wholesale price is high (i.e. $w = 12$) while the transshipment price is low (i.e., $10 \le p_t < p_1$), the manufacturer is more willing to place a smaller order quantity of the direct channel if the cooperative inventory strategy is adopted so as to encourage the retailer to order a larger amount.



a) $w = 8.1$ b) $w = 9$ c) $w = 12$

Figure 1: The comparison of each channel's order quantity under the cooperative inventory strategy and the uncooperative inventory strategy with $w = 8.1$, 9, 12.



a) $w = 8.1$ b) $w = 9$ c) $w = 12$

Figure 2: The comparison of the entire dual-channel supply chain's profit under the cooperative inventory strategy and the uncooperative inventory strategy with $w = 8.1$, 9, 12.

Fig. 2 shows that when w is moderate (i.e. $w = 9$), the cooperative inventory strategy is always beneficial to the supply chain. When the w is low (i.e. $w = 8.1$), the cooperative inventory strategy is beneficial to the supply chain if the transshipment price is low enough (i.e. $10 \le p_t \le p_2$). When the w is high (i.e. $w = 12$), the cooperative inventory strategy is beneficial to the supply chain only if the transshipment price is high enough (i.e. $p_3 \le p_t \le 20$).

However, the cooperative inventory strategy is adopted only if the cooperative inventory strategy is beneficial to each supply chain member. Let the inventory strategy choices of the manufacturer and the retailer be $(X_m X_r)$, where $X_i \in \{N, D\}$ (N – the cooperative inventory strategy, D – the uncooperative inventory strategy). Table I shows that when $w = 8.1$ or $w = 12$, the Pareto improvements of the manufacturer and the retailer are barely achieved by the cooperative inventory strategy. However, when $w = 9$, the Pareto improvements of the manufacturer and the retailer can be achieved by the cooperative inventory strategy if $p_t = 17$.

Table I: The game matrix of the manufacturer and the retailer.

p_t	$w = 8.1$	$w = 9$	$w = 12$
11	DN	DN	DN
17	ND	NN	DN
20	ND	ND	ND

The numerical examples above suggest that the profit of each supply chain member under the cooperative inventory strategy can be higher than those under the uncooperative inventory strategy. However, we need to set a reasonable transshipment price p_t and a reasonable wholesale price w and other reasonable factors to implement the cooperative inventory strategy in a dual-channel supply chain successfully.

6. CONCLUSIONS

In the paper, we put forward the inventory cooperation between the traditional retail channel and the manufacturer-controlled direct channel to deal with the risk of being out of stock in the traditional retail channel. We discussed the impacts of some relevant parameters on order quantity decisions and optimal profits under the uncooperative inventory strategy and under the cooperative inventory strategy respectively. Compared with the uncooperative inventory strategy, the cooperative inventory strategy can not only improve the profit of the entire dual-channel supply chain but also achieve the Pareto improvements of the supply chain members when the transshipment price and the wholesale price satisfy certain conditions.

In the further research, we may study the inventory cooperation issue with endogenous transshipment price and wholesale price in a dual-channel supply chain. In addition, we may also explore the case in which the information is asymmetric.

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