

HARDNESS MODELLING OF DEFORMED CW106C ALLOY BY A GENETIC PROGRAMMING

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Abstract

In the paper an evolutionary algorithm method for Brinell hardness modelling of cold deformed copper alloy is presented. Genetic programming method, described in the paper, is very powerful modelling method in the field of evolutionary algorithms. During our investigation, CW106C alloy was cold drawn on drawing bench and the impact of drawing parameters on the change of hardness of deformed alloy was determined. One part of experimental results was used as training data for genetic programming process with the main goal to obtain accurate and suitable models for hardness prediction in deformed alloy. The adequacy of genetically developed models was checked by a testing data. For a comparison a standard linear regression method for modelling is also presented in the paper. These models can be used not only to predict the material hardness but also to search for optimal process parameters for desired hardness of the formed material.

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Key Words: Cold Forming, Hardness, Alloy, Evolutionary Algorithms, Genetic Programming, Modelling

1. INTRODUCTION

Prediction and optimization of material hardness is a very important in both cold and hot metal forming processes. With more and more new materials appearing in production technology, using of existing models is not possible and because of this more experimental work is needed. Different process parameters have impact on the hardness during cold forming process, the most important being effective strain. Large number of different methods for dependent parameters prediction was developed to lower the number of needed experiments and consequently reducing the investigation expenses. Conventional deterministic methods, such as regression analysis, are often not suitable (or not accurate enough) to deal with complex parameters connections. Therefore, non-deterministic methods, such as evolutionary algorithms, are more appropriate tools for accurate modelling and prediction. Evolutionary algorithm (EA) is an artificial intelligence (AI) method which is inspired by Darwinian evolution. It is very useful engineering tool for modelling, predicting and optimization of different engineering and also non engineering problems [1]. Very important EA method is genetic programming method (GP), developed and used in our paper, which can be described as very general and powerful AI method that imitates evolutionary process in the nature.

Cold forming represents an increasingly important proportion of processing technologies. In paper [2] the distribution of contact stresses along the die-workpiece interface and elastic deformation behaviour of the die in the cold backward extrusion of steel billets have been investigated both experimentally and numerically.

In their investigations many researchers have used neural network or genetic algorithms methods for optimization and modelling of different material characteristics. In paper [3] authors describe modelling of material properties of composites by using artificial neural networks method and thin plate spline method. The results have shown that models obtained with both methods have high accuracy in prediction of hardness of the composite material. Authors in papers [4, 5] have analysed modelling of mechanical properties of different

aluminium alloys using artificial neural network and also statistical methods. Comparison between these two methods were also presented and discussed. Impact of different input variables on mechanical characteristics by using artificial neural network for modelling was described in papers [6, 7] in which the impact of casting parameters on material hardness is studied and regression models were obtained for prediction of investigated characteristics. Modelling of material hardness with genetic algorithms is presented in [8] where also some other material characteristics were investigated. Very accurate and useful mathematical models were obtained by regression method. In research papers [9-12] a neural network method for prediction of different material characteristics was presented while in papers [13, 14] GA method was developed very successful to optimize the process modelling.

Considerably fewer manuscripts are using more general GP method as a tool for modelling of mechanical and other material characteristics [15, 16]. In paper [17] authors compared models obtained with genetic algorithm and those obtained by GP approach for the distribution of stress in cold formed alloy. Both methods were very suitable in obtaining appropriate models but GP method provided more accurate genetic models. Genetically developed model was successfully applied in simulation software for bulk forming analysis. Some authors have shown the possibility of applying GA and GP to optimize prediction models in automotive industry [18] and models for material properties such as hardness [19] of the steel. Applied genetic method proofed itself as very useful method for modelling and optimisation. Influence of friction on hardness distribution in cold formed material was analysed in [20]. Different lubricants such as Teflon or grease were examined for the impact on hardness distribution during forging and extrusion. Semi solid lubrication performed better results for hardness prediction than liquid. The experimental exploration of cutting forces produced during ball-end milling of multi-layered metal materials manufactured by the laser engineered net shaping (LENS) process was presented in paper [21]. Hardness and thickness of the particular manufactured layer have been considered during training of the ANN model.

The paper describes the development and application of GP method for the modelling of Brinell hardness. Measurement results which were achieved in the process of copper alloy drawing were used as an environment for evolution process in GP. Genetically developed models must adapt to this environment as much as possible. With better adaptation the accuracy and suitability of the models will be much higher. In performed experiments input parameters were coefficient of friction and effective strain. Those parameters were independent, while Brinell hardness was a dependent parameter. The prediction models for hardness were obtained by GP method with training data. Accuracy and suitability of these genetic models were then checked by testing data, which were not included in training data.

2. GP METHOD

GP is a sophisticated extension of genetic algorithm and one of the most important techniques amongst great number of evolutionary algorithms (EA). It is more powerful modelling tool than GA but it also requires more computational time and power [22]. For successful performing of GP process specification of five preparatory tasks are required [23]:

1. Terminal set specifications,
2. Functions set specifications,
3. Defining of fitness measure (i.e. difference between experimental and genetically obtained results),
4. Control parameters,
5. Criterion for the process termination.

The scheme of these required preparatory tasks is presented in Fig. 1. The preparatory tasks must be defined and inserted to GP system by human. The result (output) of the GP

system is genetic model (computer program). The definition of terminal and function set usually depends on a specific nature of investigated problem. The function set can contain different mathematical functions such as addition, multiplication, exponent function, sinus or logarithm function, etc.

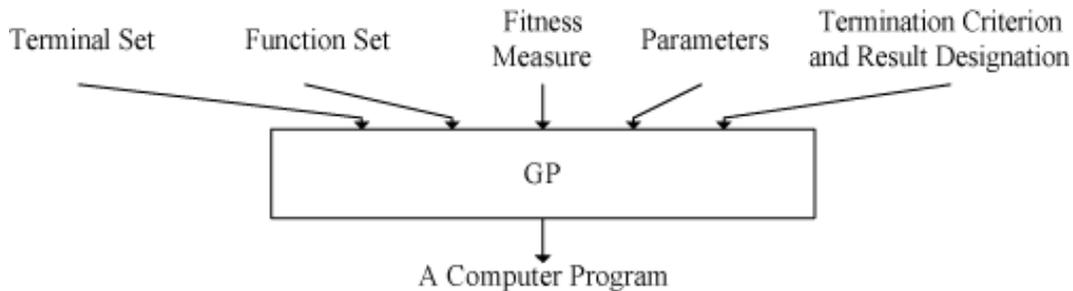


Figure 1: Major preparatory tasks for GP method [23].

Numerical constants and independent variables are usually used to form terminal set [23]. The GP process starts with creating initial population from random computer programs which are created of terminal and function genes. The third preparatory task is fitness measure calculation. This means that each of the computer programs is evaluated by trying it out on the target problem and the obtained result presents a measure of computer program (model) quality.

Control parameters, such as the population range, probabilities of the genetic operations, the depth of the models etc., are defined in preparation task number four. The selection of genetic operations and their probability is of vital importance for successful GP process. The reproduction, mutation and crossover operations provide an increasing diversity and genetic exchange among computer programs. Very few researchers use another genetic operation – permutation (inversion), because the effectiveness of using permutation operation is very questionable and therefore dismissed by a vast majority of GP users.

The last preparatory task is the definition of termination criterion. Very often a prescribed highest generation's number is selected for GP process termination criterion. When the termination criterion is reached the GP process is finished.

3. EXPERIMENTAL DETAILS

In the experimental work we analysed the influence of two process parameters on Brinell hardness of cold deformed CW106C copper alloy. First and the most important parameter was effective strain ε , while second parameter was the lubricator's coefficient of friction μ .

Table I: Chemical composition of CW106C.

Cr	Zr	Fe	Ni	P
0.71 %	0.050 %	0.006 %	0.018 %	0.001 %

In Table I the chemical composition of CW106C alloy is presented. CW106C alloy is known for its good electric and thermal conductivity and also very stable mechanical characteristics at medium and high temperatures. It is widely used in automotive and welding industry.

CW106C rods were cold drawn on horizontal drawing bench. The basic process parameters, such as the angle of the drawing die and speed of drawing process ($v = 0.33$ m/s) were constant during all performed experiments. CW106C was drawn at room temperature from starting diameter $D = 20$ mm gradually to 6 reduced diameters (with 1 mm diameter

reduction in each drawing phase). After each drawing phase sufficient number of specimens was taken for measurements purpose. Some specimens of alloy after drawing process with different reductions in diameter are presented in Fig. 2.

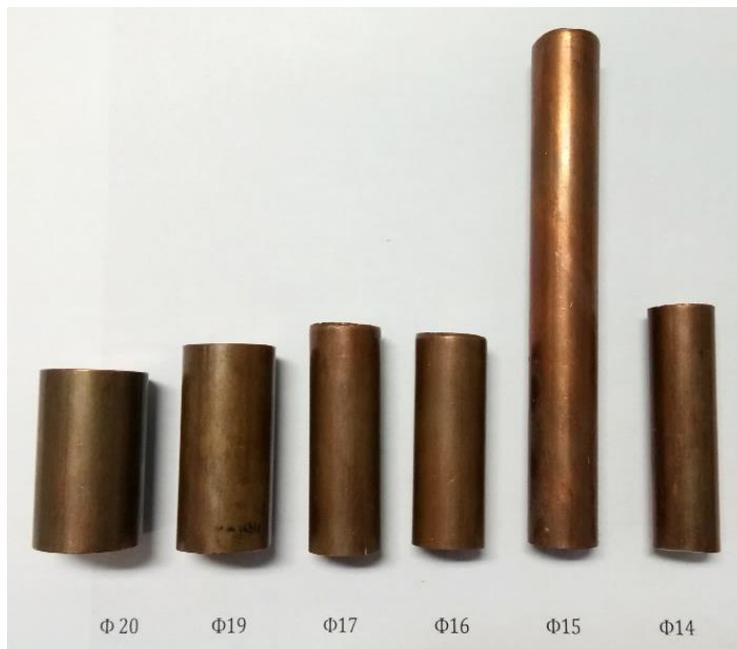


Figure 2: Alloy specimens after different deformation by drawing ($v = 20$ m/s, $\mu = 0.16$).

For lubrication purpose different lubricants were applied. Each of the three used lubricants had different coefficients of friction. The lowest friction value was measured for lubrication oil ($\mu = 0.07$), while the highest value had grease ($\mu = 0.16$). Measurements of coefficient of friction for each of the three lubricants were carried out by using the ring-compression test in which a ring-shaped billet is compressed by two plates in several increments [24]. Deformation of diameter and height was measured after each increment and by using the diagrams coefficient of friction was determined.

For each lubricant three ring tests were performed and then average values of coefficients of friction were calculated. In order to evaluate the Brinell hardness, hardness measurements were performed by the WPM hardness device. Applied force was $F = 625$ N, diameter of steel indenter $D = 2.5$ mm and time of indention $t = 15$ s.

Five measurements points on each specimen were examined for Brinell hardness and average values for each specimen were calculated from the equation where d is diameter of indentation [25]:

$$HB = \frac{2F}{\pi D(D - \sqrt{D^2 - d^2})} \quad (1)$$

Locations of hardness measurement points were the same for all specimens. 90 hardness measurements were performed and, when considered average values, 18 different experimental measurements were obtained.

The CW106C alloy hardness measurement results are shown in Table II and Table III.

The experimental results were split into two parts. 12 experiments were taken to build training data set for GP process (Table II), while other six experimental results were used as a testing set for evaluation (testing) of GP models (Table III).

Table II: Brinell hardness measurements.

Exper. No.	Rod diameter D (mm)	Effective strain ε	Coefficient of friction μ	Brinell hardness
starting	20	/	/	141
1	19	0.10	0.07	147
2	19	0.10	0.11	146
3	19	0.10	0.16	151
4	18	0.21	0.07	150
5	17	0.32	0.07	154
6	17	0.32	0.11	156
7	16	0.44	0.07	159
8	16	0.44	0.16	164
9	15	0.57	0.11	165
10	14	0.71	0.07	165
11	14	0.71	0.11	167
12	14	0.71	0.16	170

Table III: Brinell hardness measurements (GP testing data).

No.	D (mm)	ε	μ	HB
13	15	0.57	0.07	164
14	18	0.21	0.11	153
15	16	0.44	0.11	160
16	18	0.21	0.16	154
17	17	0.32	0.16	158
18	15	0.57	0.16	167

4. DEVELOPMENT OF GP MODELS FOR PREDICTING HARDNESS

GP process is an iterative procedure with clearly visible contours of evolutionary algorithms code: generation of initial population of solutions, analysis and evaluation of every organism (model) in the population, and selection of individuals for evolutionary operations, usually by tournament scheme. Genetic operations were applied for creation of new organisms in new generation. The best genetic model with the highest fitness in the population was a final result of GP method. A number of preliminary GP runs were executed with the goal to find out the most appropriate values for evolutionary parameters like crossover, mutation and reproduction for ensuring optimal GP process and best modelling results. As a process termination criterion the maximum allowed number of generations was determined. The applied evolutionary parameters for GP modelling process were:

- population size: 1000,
- maximum allowed generations number: 100,
- reproduction probability: 0.2,
- mutation probability: 0.1,
- crossover probability: 0.7,
- maximum depth of organisms in initial generation: 7,
- maximum depth of organisms after crossover: 12,
- highest depth of segment for mutation: 7.

Modelling of hardness with GP method was performed in two steps. In the first step each GP run started with the training data (Table II). Every run temporarily stopped when

generation 50 was reached to control the accuracy of genetic models in the population. When the population contained one or more genetic models with the accuracy better than 5 %, it was allowed to continue the evolution process till the end. Populations which didn't fulfil this criterion were terminated. In the second step, the testing data (Table III) was used to control the accuracy of the best genetic models. In our research, for each individual function set 100 independent runs were performed.

4.1 Best developed genetic models

Two function sets were used for GP modelling. First function set contains four mathematical operations: (+, -, *, %). In the second function set exponent function was added to other four operations (+, -, *, %, ZEXP).

When using the first function set the genetic model with the best accuracy was calculated in 97th generation. The LISP form of this model is:

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((+ (% μ (+ (+ (* (- (+ 3.5704 8.12616) -1.30479) μ) (+ (- ε ε) (+ μ -6.4277))) (* (* -9.91045 (- (+ (% -4.47358 ε) (* ε 3.02234)) (* (- (+ 2.71631 ε) μ) (* μ μ))) (* μ ε))) (+ (+ (- (- (* (+ (* μ ε) (+ (-ε -5.95963) ε)) 8.39704) (* μ ε)) (+ (* μ μ) (+ μ (-ε ε)))) (% (* (+ε μ) (- (- (+ 2.71631ε) μ) (- -8.04343 ε))) (+ (% -4.47358 ε) (* ε 3.02234)))) (- (* (+ (* μ 5.90816) (- 8.41076 μ)) (- (- (+ 2.71631 ε) μ) (- -8.04343 ε))) (% (% (+ -0.554034 7.49166) (+ (% -4.47358 ε) (+ 6.71361 ε))) (+ (-ε ε) (+ 6.71361 μ))))))
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Symbol % in genetic programming means protected division operation (to prevent division by zero). Mathematical expression of this model:

$$\frac{50.043 - \mu - \varepsilon\mu - \mu^2 - \frac{6.937\varepsilon}{(\varepsilon^2 + 6.713\varepsilon - 4.473)} + \frac{(10.759\varepsilon + 2\varepsilon^2 - \mu)(\varepsilon + \mu)}{3.022\varepsilon^2 - 4.473} + (10.759 + 2\varepsilon - \mu)(8.41 + 4.908\mu) + \varepsilon(16.794 + 8.397\mu) + \mu}{-6.427 + 58.336\mu - 29.952\varepsilon^2\mu + 26.919\varepsilon\mu^3 + 9.91\varepsilon^2\mu^3 - 9.91\varepsilon\mu^4} \quad (2)$$

Average calculated difference between experimental and genetic results for this model is $\delta = 0.18$ % when training data were used and $\delta = 0.20$ % for testing data.

Bigger error ($\delta = 0.28$ % for training data, and $\delta = 0.30$ % for testing data) was calculated with the best genetic model developed with the second function set with added exponent function:

$$e^{-e^{\mu-\varepsilon-8.646}} \left(\frac{\varepsilon+\mu}{2088.35 + e^{10.163e^{-2\varepsilon}} - 1.383e^{2\varepsilon} \varepsilon + \frac{1.019e^{-\varepsilon}}{\varepsilon - \mu}} \right) \quad (3)$$

Some very simple genetic models were also obtained by GP but the deviation of these models was not good. The most accurate of all simple GP models was obtained with basic operations (+, -, *, %):

$$136.05 + 29.805 \varepsilon + \varepsilon^2 \quad (4)$$

This model has the depth of 5, consists of 8 genes and the calculated average error $\delta = 3.8$ % (testing data set $\delta = 4.2$ %) which of course is not good enough especially when compared to models (2) and (3), but the model is very simple in its structure and easy to use. It is also interesting that in GP model (4) the evolution process has excluded the coefficient of friction and the final genetic model contains only effective strain as independent variable.

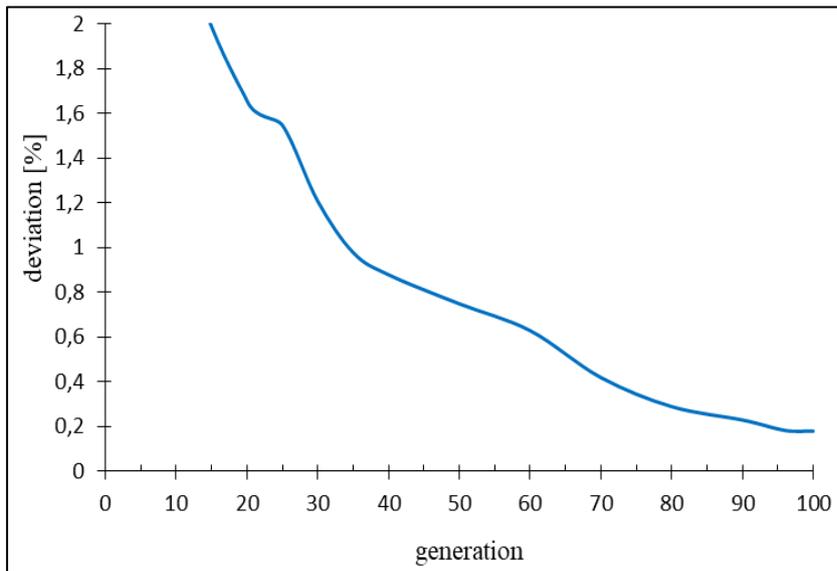


Figure 3: Deviation of the best genetic models in the most successful run.

The deviation (error) of the genetic model in the most successful run obtained with the first function set (+, -, *, %) is presented in Fig. 3. The form of the curve shows that in first 12 generations deviations of genetic models reach more than 2 %; model obtained in generation 13 is the first model with deviation less than 2 %. From generation 25 to generation 40 there is a large improvement of accuracy of genetic models. After that the accuracy of best models increases very slowly especially from generations 40 and up to 62.

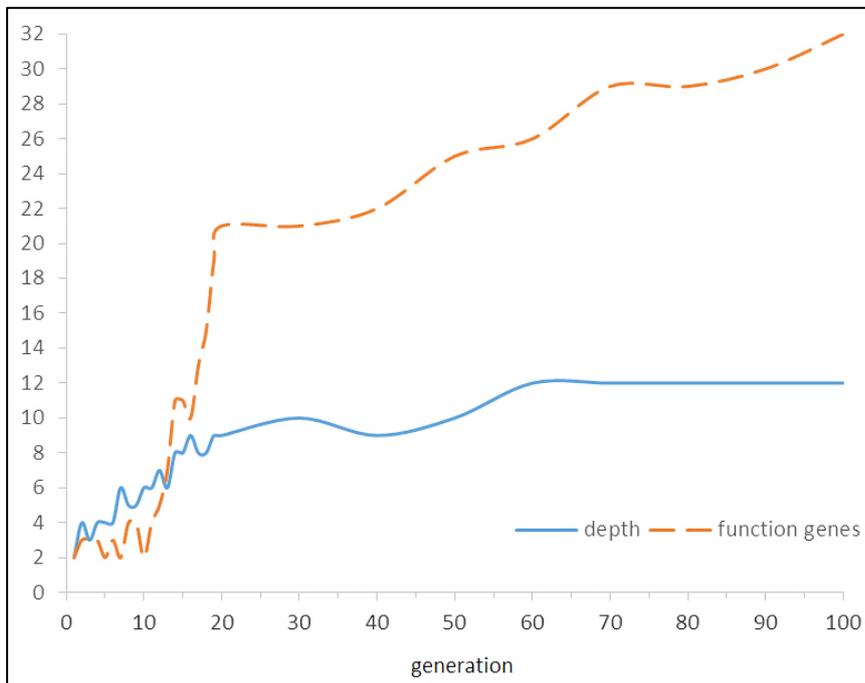


Figure 4: The depth and number of function genes of best GP models for the most successful run.

The depth and number of function genes of best obtained genetic models in the most successful run for every generation by applying the first function set is presented in Fig. 4. There is a great dynamic in both, depth and number of function genes, from first and up to 17th generation. The number of function genes alternately increase and decrease during early generations but after generation 17 the function genes constantly increase and reach the value

21 in generation 20 and 32 in the last generation. Depth of the models also varies at the beginning and reaches the depth of 10 as early as in generation 22. Between generations 22 and 59 the best GP models have a depth between 9 and 11. Finally, in generation 60, the model depth reaches a maximum of 12 and all of the best GP models up to final generation have constant depth of 12.

5. REGRESSION MODEL

For comparison a multi linear regression method was used for modelling the hardness. The multiple linear regression is an extension of a simple linear regression to incorporate two or more explanatory parameters in a prediction equation for a response variable [26]. It is one of the most often used statistical analysis because of its power and flexibility.

By inserting the values of coefficients obtained by SPSS program, the regression model for Brinell hardness can be presented as:

$$HB = 141.158 + 53.963 \varepsilon - 42.485 \mu - 24.186 \varepsilon^2 + 450.159 \mu^2 - 16.305 \varepsilon \mu \quad (5)$$

This mathematical model can be used for prediction of the impact of two independent parameters (ε and μ) on the hardness in the frame of experimental results. The calculations showed that the average deviation of model (5) is $\delta = 0.57\%$ ($\delta = 0.68\%$ when testing data was used). Comparison between best obtained GP models and regression model (5) shows that genetic models are more complex than regression model. Because of evolutionary nature, best GP models very often have lots of genes and their form is not simple. But when we compare the accuracies, genetically obtained models have better accuracy than models obtained by regression method.

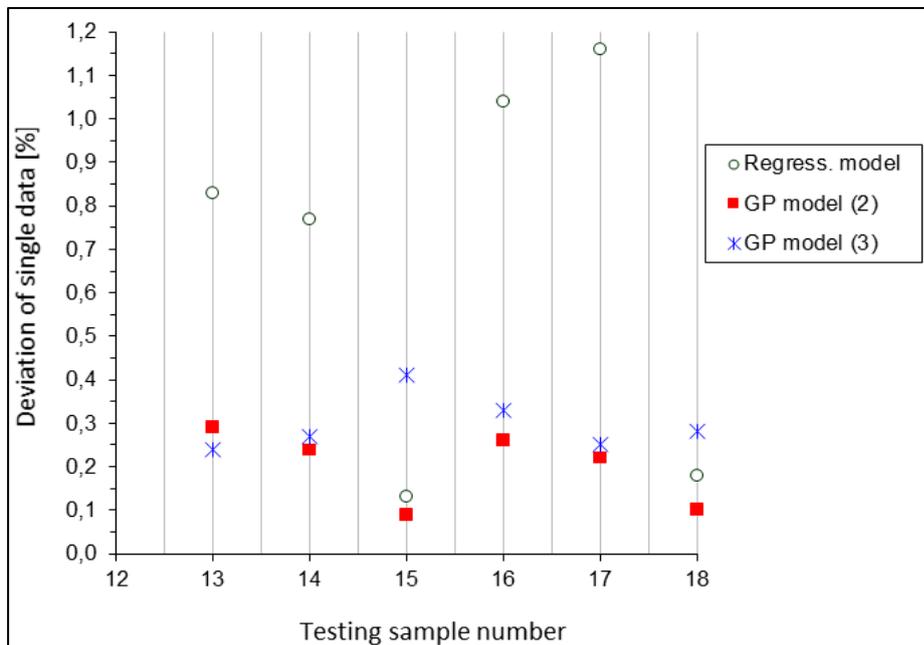


Figure 5: Absolute deviation of GP and regression models in single testing data.

Fig. 5 presents an absolute deviation between experimental results and results obtained by regression model (5), GP model (3) and GP model (2) in all testing data. Testing samples on x-axis are numbered according to the Table III. The GP models have much higher accuracy (lower deviation), in fact in every point of testing data GP model (2) has better accuracy than regression model while GP model (3) has lower accuracy compared to regression model only in two testing data.

6. CONCLUSION

The paper presents GP method for the modelling of Brinell hardness of cold formed CW106C alloy. Experimental results showed that effective strain has very great impact on hardness change while the impact of lubricant's friction coefficient is quite small. Obtained experimental results were divided into two data sets which were then used as an environment for GP process evolution. A great number of GP runs with different genetic parameters were executed. GP method proofed itself as a very successful and efficient modelling tool for obtaining very accurate models for hardness even with relatively small number of experiments. In our research the best genetic models and model obtained by multiple regression were compared for their accuracy and complexity. The comparison showed that best genetic models have much better accuracy but are also more complex than regression models.

With the proposed GP method, change of hardness in formed alloy can be predicted with very high accuracy. Presented GP models are also very useful for optimization of process parameters for the desired hardness of formed material. In the future work we are planning to optimize the parameters of GP method to obtain even more accurate genetic models for prediction of hardness and other properties of formed materials.

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