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SIMULATION OF INTENSIVE CARE BED CAPACITY BASED ON MIXTURE DISTRIBUTION

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Abstract
Intensive care units (ICUs) are one of the most important elements of hospitals. ICUs play a central role in the healthcare system and in providing care for critical patients, so capacity planning in these units is critical. A shortage of ICU beds and staff can have irreparable consequences, including patient death. As a result, hospital managers make efforts to determine the appropriate number of beds. However, the interarrival time (*IAT*) of patients to ICUs and the service time (*ST*) of patients in ICUs are stochastic in nature. Consequently, capacity planning is a dynamic operations management problem. For this research, we used mixture distributions to approximate the interarrival time (*IAT*) and service time (*ST*) of patients in ICUs. We then incorporated these distributions into a simulation model that helps us to determine the number of beds needed to accommodate all incoming patients without any waiting in the queue. The results show that the mixture distributions provide a better estimate than empirical statistical distributions.
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Key Words: Length of Stay, Interarrival Times, Mixture Distribution, Bed Capacity, Intensive Care Unit

1. INTRODUCTION

Intensive care units (ICUs) play an important role in the healthcare system, providing care for patients who require immediate attention [1]. The number of ICU beds is one of the most important factors that directly affects the health status of patients. A shortage of ICU beds can lead to a delay in referral, jeopardising the patient's chance of a full recovery or, worse, the patient's life [2]. Given these critical conditions, the demand for ICU beds must be adequately met. Given the stochastic nature of demand for ICU services in hospitals, resource allocation is a challenging and dynamic problem.

Decision makers often use quantitative analysis techniques to solve dynamic problems. Simulation modelling is one of the best methods for solving dynamic resource allocation problems with constraints [3]. The ICU consists of three elements: interarrival time or arrival rate, length of stay or service time, and the number of servers [1]. Understanding the structure of the distribution of the interarrival time (*IAT*) and service time (*ST*) of ICU patients can support operational decision making in hospitals. The accuracy of the simulation model depends on well-predicted input parameters. However, fitting the pure statistical distribution of *IAT* and *ST* is challenging because the data are generally random, highly right-skewed, and long-tailed distributions [4].

Weibull, gamma, and lognormal distributions, which are commonly used to fit *IAT* and *ST*, can represent this skewness and tail behaviour [5]. However, these pure statistical distributions do not fit the *IAT* and *ST* of patient populations with a significant number of outliers [6]. In such cases, researchers have generally preferred the empirical distribution. Empirical distributions contain less information than pure statistical distributions. To deal with this limitation, the mixture distribution has been proposed instead of the approximated empirical distribution [7, 8]. Despite the proposed use of mixture distribution models as an alternative to

approximated empirical distributions in healthcare, only a small number of researchers have utilized this method to analyse health systems with data that do not conform to any pure statistical distribution [9].

In this study, the use of mixture distribution models is presented for the analysis of highly skewed and long-tailed Interarrival Time (*IAT*) and Service Time (*ST*) distributions. These mixture distributions are flexible and do not require outlier removal or data transformation. However, the calculation of the parameters of the distributions in a mixture distribution model requires complex maximum likelihood estimation algorithms, such as the Newton–Raphson or expectation maximization (EM) algorithm. Despite these limitations, mixture distribution models have been proposed as a useful modelling technique to understand the *IAT* and *ST* of patients. In this study, normal, Weibull, gamma, and lognormal mixture distribution models are proposed for fitting the *IAT* and *ST* data and the EM algorithm is used to estimate the mixture distribution model parameters.

Since ICU patients require immediate advanced healthcare support, the simulation model of this paper aims to find the number of ICU beds that can serve all incoming patients without waiting times in the bed queue. Our aim in this study is to develop comprehensive input parameters using mixture distributions for *IAT* and *ST*, which are generally highly skewed and contain a significant number of outliers, and to integrate these estimated mixture distribution parameters into the simulation model.

In summary, the research objectives of this paper are as follows:

- To simulate the process for patients in two ICUs.
- To predict the most appropriate *IAT* and *ST* of patients using mixture distributions and integrate these parameters into the simulation model.
- To find the number of beds that ensures no waiting times for patients.
- To investigate process outputs such as the percentage of rejected patients, bed utilization, and minimum average waiting times.

The paper is organized as follows. Section 2 provides a comprehensive review of the relevant literature. Section 3 outlines the research methodology, which encompasses the identification of the system, the description and fitting of the data distribution, the presentation and fitting of mixture distributions, the calculation of the EM algorithm required for parameter estimation, the validation of estimated parameters, and the development of the simulation model. Section 4 presents the research findings, which are founded on both real and simulation-based approaches. Lastly, Section 5 discusses the outcomes and implications of the study and suggests potential avenues for future research.

2. LITERATURE REVIEW

The literature review presented in this section is divided into two subsections. In the first subsection, the review focuses on the use of simulation modelling in healthcare systems. The second subsection focuses on the use of distribution models of input parameters in simulation models.

2.1 Simulation modelling

This subsection examines the studies that have used simulation models to analyse performance and resource allocation in healthcare systems. Health capacity management in the healthcare system involves determining the necessary resources, such as staff, beds, and medical supplies, that are required to enhance the performance of the system. These systems, with their numerous variables and limitations, necessitate the use of complex linear and integer mathematical models to accurately depict and solve the real system's problems. Although there have been reports in the medical literature of discrepancies between assumptions in mathematical simulation models

[10], this method has still been widely adopted by various authors over recent years for capacity management in the healthcare system [4, 11]. Studies examining the application of simulation models in healthcare can be found in the literature, including [12, 13].

The complexity of the approximated mathematical model can be captured by the simulated data and a simpler model can be adopted [14]. Simulation models have been widely used to suggest policies based on different outcomes and probable or best scenarios in health systems [15]. The aim of simulation models is to meet the objective function and to detect the number of optimal resources [16].

The authors of one paper combined healthcare system simulation with optimisation to determine the optimal number of doctors and nurses in an emergency department [17]. Aside from capacity planning, a significant number of studies have also addressed the problems of the design of healthcare systems of healthcare clinics by means of simulation and optimisation tools. In addition to staff planning, the literature has also addressed the problem of consulting room design using simulation and optimisation tools [18].

2.2 Input parameter estimation

The aim of this subsection is to provide a comprehensive overview of the application of distribution models in healthcare simulation and to highlight the importance of using realistic input parameters for the simulation models.

Simulation input parameter estimation in healthcare refers to the process of determining the values of inputs for a simulation model that is used to study various aspects of the healthcare system. In the last decade, various computational methods have been employed to predict *IAT* and *ST*. Machine learning methods have been utilized in some studies [19, 20], as well as time series models to predict patient arrival [21]. However, there remains a gap in the research, as these methods may not fully represent the system and rely on unrealistic homogeneity assumptions. Additionally, the skewed and outlier nature of *IAT* and *ST* distributions means that it is challenging to fit them into pure statistical distributions.

Recently, the use of mixture distribution models has emerged as a promising solution for dealing with skewed data and supporting decision making in healthcare management. The incorporation of mixture distribution models into a simulation can improve the representation of input parameters, making the queuing model a more accurate representation of real-time conditions. Mixture distribution models have been applied in a variety of fields, including business and marketing, as well as various branches of healthcare such as anatomy, bioinformatics, and cell biology [22]. *IAT* and *ST* data have been fitted to normal, negative binomial, exponential, Weibull, Poisson, and lognormal mixture distributions in some studies [7, 23-25]. According to the literature review, in this study, four different continuous mixture distributions (normal, Weibull, gamma, and lognormal) are implemented.

This paper aims to further advance the role of mixture distribution in fitting *IAT* and *ST* by developing a comprehensive model that utilizes a simulation approach, incorporating a mixture distribution model as an input parameter. The literature review highlights that this work represents a novel approach for accurately estimating the required bed capacity by utilizing a mixture distribution model to represent real data more effectively than empirical distributions.

3. METHODOLOGY

3.1 System description

The entities in the model are patients who arrive at random intervals. The process commences upon the patient's arrival at the ICU. The flow of events is depicted in Fig. 1, which outlines the procedure for patients arriving at the ICU. The time of arrival for each patient is recorded

in the hospital database. The first come, first served (FCFS) policy is used to decide the order in which patients occupy the beds.

The objective of the simulation model used in this study is to determine the number of beds required to serve all incoming patients without any waiting time in the queue. In the event that all beds are occupied when a patient arrives at time t, the patient will leave the system without receiving treatment. On the other hand, patients who are able to secure a bed will occupy it for the specified service time and then depart from the system.

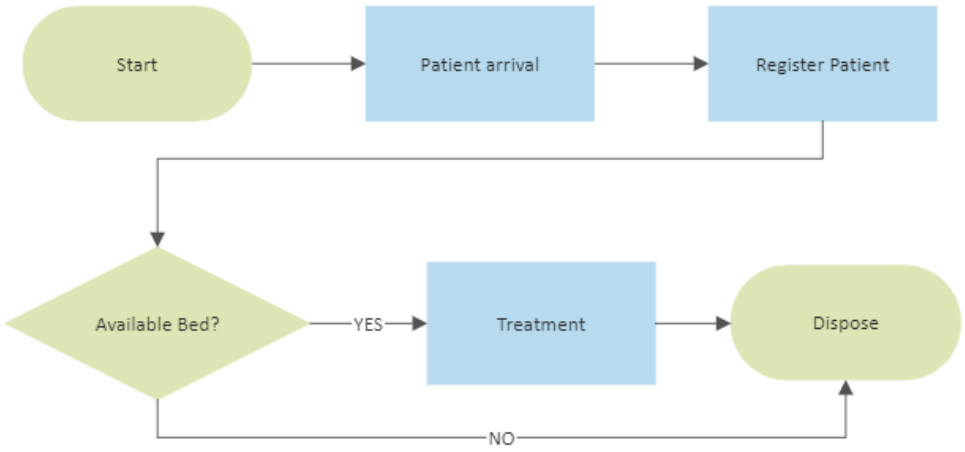

Figure 1: Representation of patient-flow logic in the ICU system.

3.2 Data collection

The data utilized in this study were sourced from the database of a teaching hospital located in Adana, Turkey, which offers round-the-clock medical care. This paper focuses on the patient flow in two departments, specifically the Pediatric ICU and the Reanimation ICU. The values of *IAT* and *ST* pertain to patients who were hospitalized from January 2018 to December 2018 and are expressed in hours.

The basic descriptive statistics of the *IAT* and *ST* for each department are presented in Table I. The table includes sample size (n), mean (μ), standard deviation (σ), median, skewness, and kurtosis. The statistical analyses in this paper were carried out using RStudio x64 4.1.2, and the simulation models were executed in ARENA on a device equipped with an Intel Core i5-6200U@2.3GHz processor and 8 GB of RAM.

Table I: Basic descriptive statistics.

ICUs	Type	Size (n)	Mean (μ)	Std. dev. (σ)	Median	Skewness	Kurtosis
Pediatric	*IAT*	758	10.59	10.68	6.77	1.99	6.77
	ST	759	115.63	162.27	46.03	2.73	8.21
Reanimation	*IAT*	254	32.93	39.19	23.71	2.8	10.35
	ST	255	251.03	276.62	126.15	1.48	1.68

The mean and median are measures of central tendency, while the standard deviation, kurtosis, and skewness are measures of variability. If the value of kurtosis is greater than +1, the distribution is considered to be sharply peaked. A skewness within the interval of -1 and +1 is considered indicative of a normal univariate distribution, while a value outside of this interval suggests a highly skewed and asymmetric distribution. When the data are symmetrically distributed, the mean and median should be approximately equal. The results in Table I show that the values of skewness and kurtosis for both ICUs fall outside of the acceptable intervals. Furthermore, the mean and median values suggest that the *IAT* and *ST* in both ICUs are characterized by asymmetric distributions.

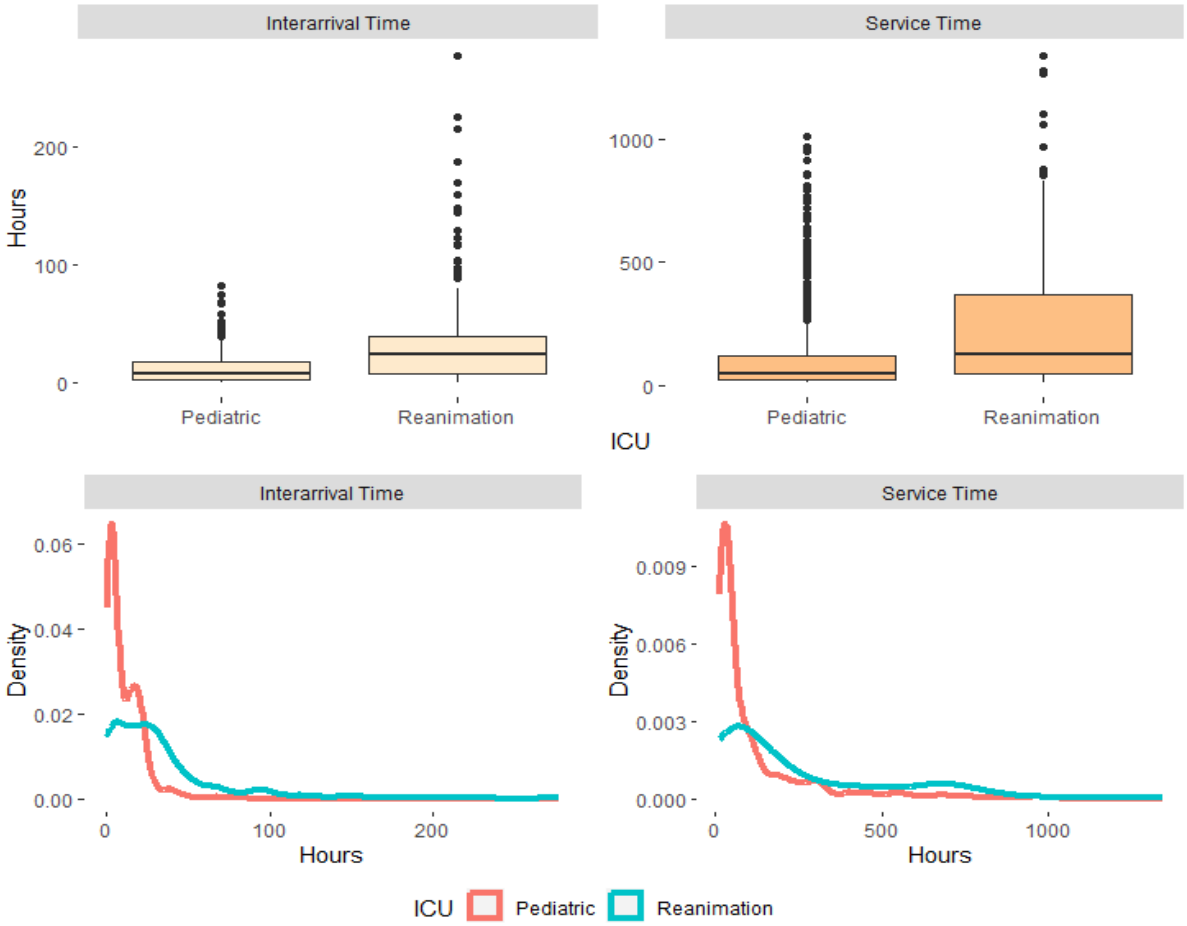

Figure 2: Boxplots and density functions of *IAT* and *ST* in ICUs.

Fig. 2 displays both boxplots and density functions. The *IAT* and *ST* values are all positive, and the density functions exhibit long tails, indicating that the *IAT* and *ST* distributions in both ICUs are right-skewed. The density functions emphasize the skewness and asymmetry of the distributions. Boxplots provide a visual representation of skewness and outliers. As depicted in Fig. 2, all values of *IAT* and *ST* are right-skewed and contain outliers.

3.3 Distribution fitting

In general, the distributions of *ST* are commonly fitted by the gamma, lognormal, and Weibull distributions [7]. The Weibull distribution is formulated as $f(x;k,\lambda)$, where k represents the shape and λ the scale parameter. The gamma distribution is represented as $f(x;k,\theta)$ or $f(x;\alpha,\beta)$, where k denotes the shape and θ the scale parameter, or where α is equal to k as the shape and β equal to $1/\theta$ as the rate parameter. When the value of k is equal to 1, the gamma and Weibull distributions are reduced to exponential distributions, making them both generalized forms of the exponential distribution. The lognormal distribution is positively skewed and exhibits a long right tail, which is obtained through the logarithmic transformation of the normal distribution. As a result, the lognormal distribution always takes positive values; the normal distribution is symmetric but the lognormal distribution is not. Both distributions are expressed as $f(x;\mu,\sigma)$, with μ indicating the mean and σ denoting the standard deviation parameter.

The Chi-squared (χ^2) test is a statistical hypothesis test used to evaluate the conformity of a variable to a specified distribution. The sample data are considered to be consistent with the null hypothesis (H_0) when it fits the hypothesized distribution. If the p-value from the χ^2 test is below the significance level (α), the null hypothesis is rejected [26]. Table II indicates that

the data do not conform to any of the five selected distributions (normal, Weibull, gamma, lognormal, and exponential), as evidenced by the significance level (α = 0.05).

Table II: Chi-squared (χ^2) goodness of fit test statistical p-values.

ICUs	Type	Normal	Weibull	Gamma	Lognormal	Exponential
Pediatric	*IAT*	0.00	0.00	0.00	0.00	0.00
	ST	0.00	0.00	0.00	0.00	0.00
Reanimation	*IAT*	0.00	0.00	0.00	0.00	0.04
	ST	0.00	0.00	0.00	0.00	0.00

If the data do not follow a pure statistical distribution, one approach would be to create an independent empirical distribution from a given sample. In this paper, empirical distributions were generated using the EnvStats library in RStudio.

In cases where the data do not fit a pure statistical distribution, the other approach is to develop a mixture distribution to represent the data. The literature recommends using a mixture distribution instead of a pure statistical distribution to obtain better results [7]. Mixture distributions refer to statistical models that combines two or more probability distributions to represent a single, overall distribution [27]. A mixture distribution is defined as a weighted combination of two or more pure statistical distributions. Each component distribution can be any type of probability distribution, such as normal, Poisson, exponential, etc. The mixture distribution model was first proposed by Pearson [28]. The mixture distribution is defined by three components: the mixture ratios ($\pi_1, \pi_2, \dots, \pi_k$), the number of clusters (k), and the distribution parameters $f_1(x;\theta_1), f_2(x;\theta_2), \dots, f_k(x;\theta_k)$:

$$f(x) = \pi_1 f_1(x;\theta_1) + \pi_2 f_2(x;\theta_2) + \cdots + \pi_k f_k(x;\theta_k) \quad (1)$$

$$0 \le \pi_i \le 1 \qquad i = 1, 2, \dots, k \qquad \sum_{i=1}^{k} \pi_i = 1 \quad (2)$$

A general representation of the mixture distribution is given by Eq. (3).

$$f(x,\psi) = \sum_{i=1}^{k} \pi_i f(x;\theta_i) \quad (3)$$

The first challenge in mixture distributions is to determine the number of components, noted by k, and to choose the best fitting model. The main goal in finding the optimal value of *k* is to partition the data into well-separated clusters. In other words, it is to divide the heterogeneous data set into homogeneous clusters. Akaike Information Criteria (AIC) and Bayesian Information Criteria are widely used to select the optimal k by balancing model accuracy and complexity [24, 27]. AIC is defined as a function of the likelihood $L(\psi)$, of the k-component model and is calculated as $2k - 2lnL(\psi)$ [29]. BIC is computed as $-\ln(n)\, k - 2lnL(\psi)$, using a stronger penalty than the AIC, which depends on the sample size, denoted as n [30]. k is chosen according to the smallest AIC and BIC values. According to the work of Keribin [31], the BIC provides a more accurate estimate than the AIC for the model selection of mixture distributions. Therefore, in this paper, BIC values were calculated to select the best fitting model of a mixture distribution, and BIC values were considered when selecting the number of components in the mixture distribution.

Figs. 3 and 4 demonstrate the BIC values of normal, Weibull, gamma, and lognormal distribution models for $k = 2, 3, 4, 5$, and highlight the best-fitting distribution models. In the pediatric ICU, the best-fitting mixture distribution for *IAT* was determined to be a three-component gamma distribution, while the best-fitting distribution for *ST* was determined to be a three-component lognormal mixture distribution, as shown in Fig. 3. On the other hand, in the reanimation ICU, the two-component gamma and two-component lognormal distributions were selected for *IAT* and *ST*, respectively, based on the lowest BIC values, as depicted in Fig. 4.

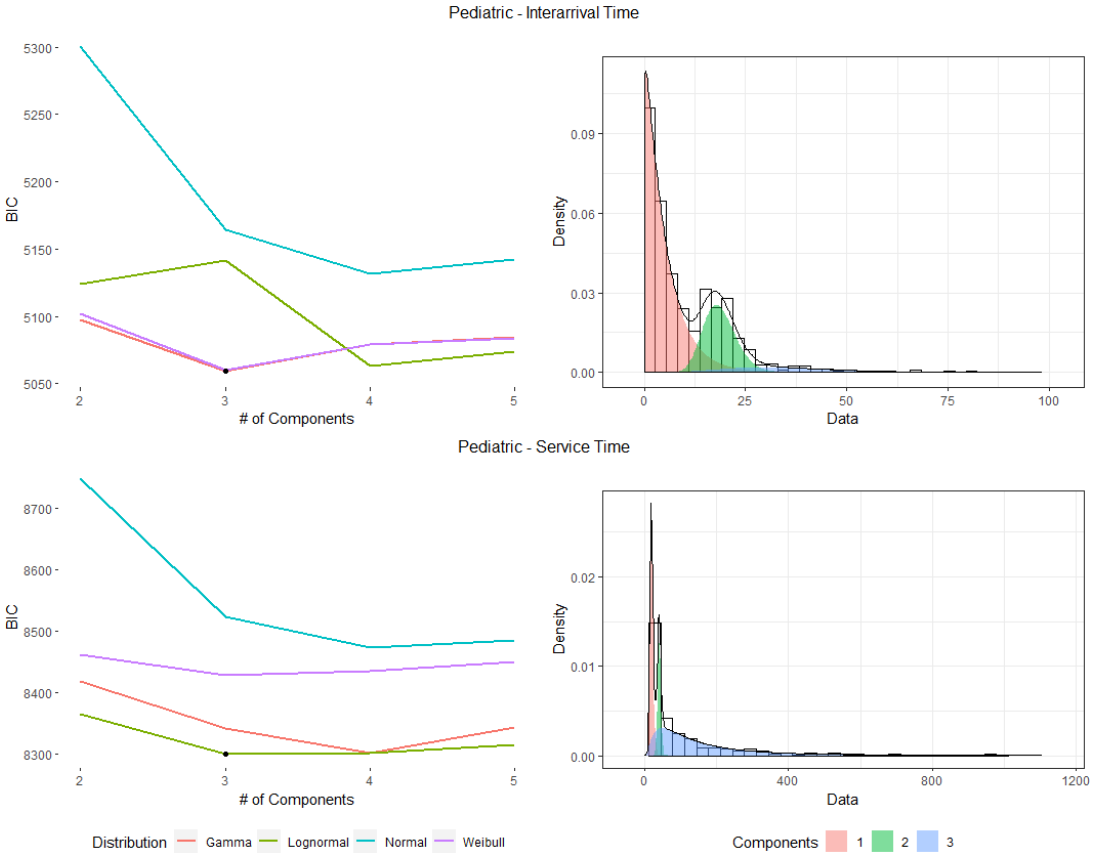

Figure 3: BIC values and best-fit *IAT* and *ST* mixture distributions of pediatric ICU.

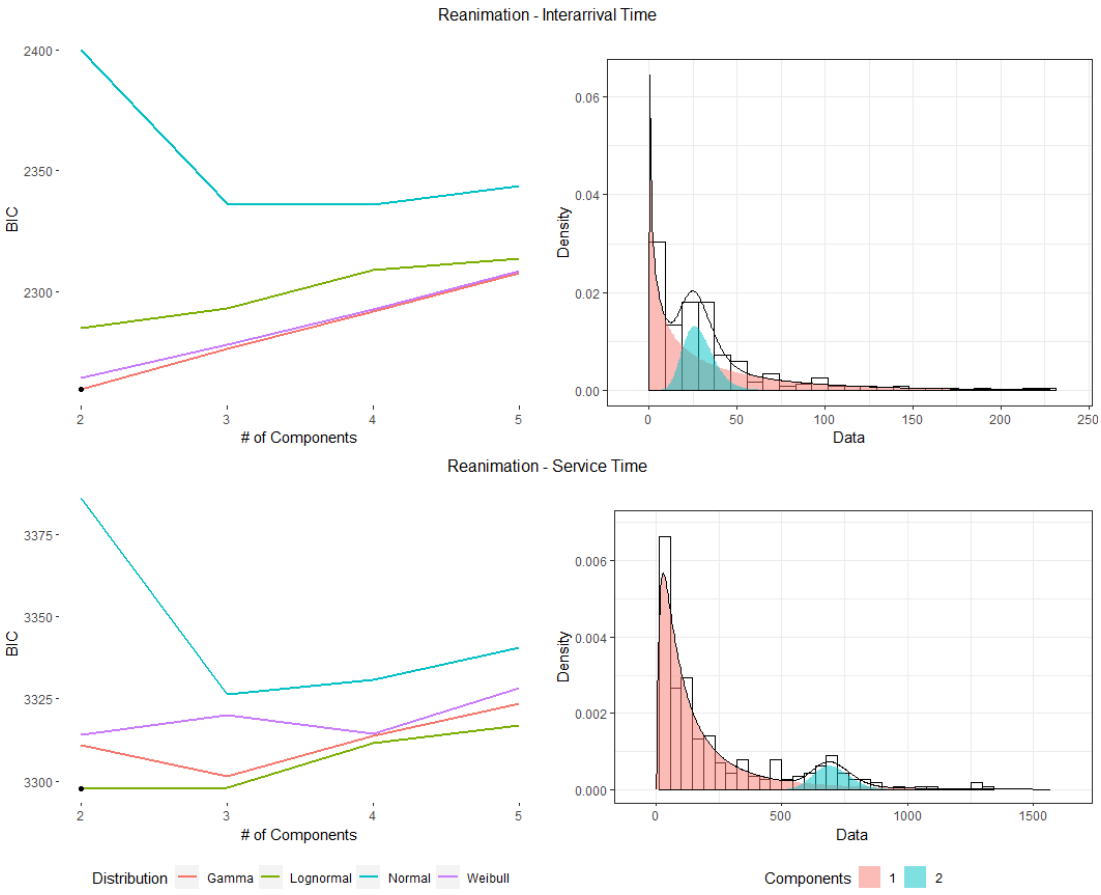

Figure 4: BIC values and best-fit *IAT* and *ST* mixture distributions of reanimation ICU.

The expectation-maximization (EM) method is a statistical algorithm that endeavours to find the parameters of a dataset that maximize the likelihood function. This approach was first introduced by Dempster [32]. The maximum likelihood estimate of the parameters $\theta(x_1, \dots, x_n)$ in a mixture distribution model with k components, created from n observations, is obtained by maximizing $L(\psi)$ for all values of θ. The components of the selected mixture distribution (the mixture ratios $(\pi_1, \pi_2, \dots, \pi_k)$, the number of clusters (k) and the distribution parameters $f_k(x; \theta_k)$) are shown in Table III.

Table III: Estimated parameters of best-fitted mixture distributions.

ICUs	**Type**	**Mixture model**	**Parameters**			
			k	π_k	θ_k	
Pediatric	*IAT*	Gamma	3		***Shape (α)***	***Rate (β)***
				0.68	1.07	0.21
				0.25	21.33	1.13
				0.07	5.96	0.16
	ST	Lognormal	3		***log-mu***	***log-sd***
				0.29	2.99	0.22
				0.15	3.72	0.11
				0.56	4.78	0.93
Reanimation	*IAT*	Gamma	2		***Shape (α)***	***Rate (β)***
				0.72	0.55	0.02
				0.28	10.63	0.37
	ST	Lognormal	2		***log-mu***	***log-sd***
				0.89	4.65	1.13
				0.11	6.54	0.11

In a pediatric ICU, the majority (56 %) of patients are discharged after a prolonged stay, as indicated by the mixture distribution of *ST*. In contrast, the service time of patients in a reanimation ICU can be represented with only two components, and the majority (89 %) of patients are discharged after a short stay. In both ICUs, the majority of patient *IAT*s are brief.

3.4 Distribution validation

Distribution validation is the process of assessing whether or not a predicted empirical and mixture distribution is appropriate.

In this paper, two methods are employed to assess the performance of the empirical and mixture distribution models: the mean absolute percent error (*MAPE*) and the Mann–Whitney U (MWU) tests. *MAPE* represents the percentage error between the actual (y_i) and predicted values $(\hat{y}_i)$ and is calculated as $100/n \sum_{i=1}^{n} |y_i - \hat{y}_i / y_i|$. A *MAPE* value greater than 10 % is considered a highly accurate prediction [33]. The MWU test is a non-parametric statistical test that determines whether two samples of quantitatively scaled observations are from the same distribution. The H_0 hypothesis is that the distributions of the two populations are identical. If the p-value of the MWU test is less than or equal to the significance level (0.05), the H_0 hypothesis is rejected [34].

The *MAPE* values for the mixture distribution approach are all below 10 %, so it is deemed to be a highly accurate prediction. Conversely, the *MAPE* values for the empirical distribution range from 9 % to 22 % and can be considered to have a low level of accuracy. In all cases, the mixture distribution approach had p-values for the MWU test that were above the significance level (0.05), implying that the H_0 hypothesis of identical distributions between the two populations was not rejected.

Table IV: Performance metrics of empirical and mixture distributions.

ICUs	Type	Empirical distribution		Mixture distribution	
		MAPE (%)	***MWU***	***MAPE*** (%)	***MWU***
Pediatric	*IAT*	16.08	0.48	8.21	0.84
	ST	9.11	0.24	3.52	0.93
Reanimation	*IAT*	22.71	0.49	8.85	0.89
	ST	19.89	0.14	8.31	0.83

The results of the distribution fitting and validation demonstrate that, when data do not conform to a pure statistical distribution, a mixture distribution approach can provide a more realistic model than empirical distribution. Consequently, the estimated parameters of the mixture distribution model for *IAT* and *ST* in the ICUs can be utilized as input parameters in a simulation model.

3.5 Simulation model

The objective of the simulation model presented in this paper is to determine the number of ICU beds required to meet the demand of all patients without incurring any waiting time. The simulation model was implemented using ARENA, as illustrated in Fig. 5.

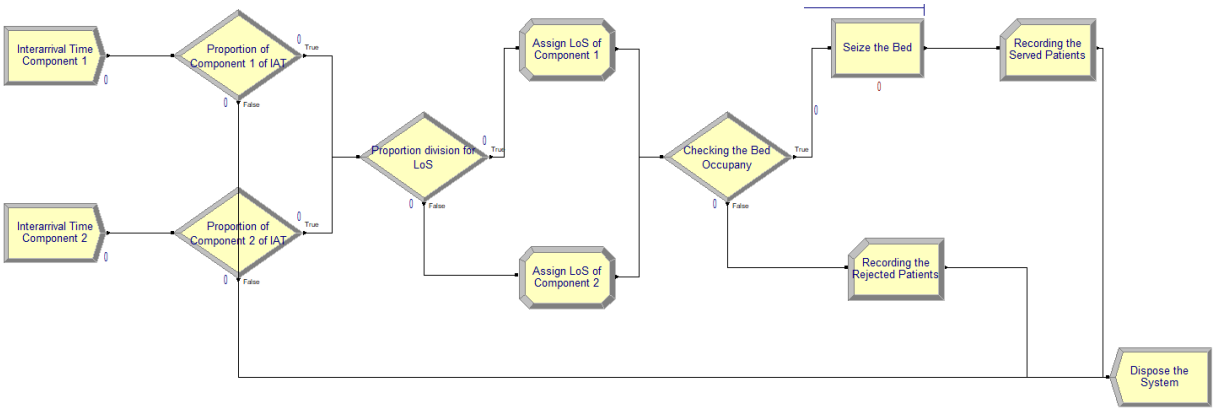

Figure 5: Interface from ARENA.

Determining the Number of replications

The reliability of the simulation outcomes was enhanced by conducting multiple runs of the simulation model. The common techniques for determining the appropriate number of replications are the fixed sample size method and the sequential method. In this study, the sequential method was utilized to establish the number of replications, and it was determined that 50 replications were adequate.

Determining the Warm-up period

In the simulation setup, a warm-up period of 876 hours, which is 10 % of the total simulation duration of 8760 hours (equivalent to 1 year), is set prior to the collection of results. This approach aligns with the commonly accepted practice of setting the warm-up period to 10 % of the total simulation duration [35].

4. RESULTS

The aim of this section is to examine the performance of the actual system under two distinct scenarios and a simulation-based approach as a third scenario. The results are presented in Table V, which demonstrates the percentage of the utilization of beds, the percentage of

rejection, and the minimum average waiting times in the two ICUs under different scenarios. Scenario 1 involves the queuing of patients in the presence of bed occupancy and service provision as soon as a bed becomes available; meanwhile, in Scenario 2, patients are transferred to alternate healthcare facilities in the instance of bed unavailability. The present quantity of beds in the pediatric and reanimation intensive care units is 13 and 8.

Table V: Simulation results in different scenarios.

	Scenario 1		**Scenario 2**		**Scenario 3**	
	Pediatric	**Reanimation**	**Pediatric**	**Reanimation**	**Pediatric**	**Reanimation**
% of rejection	0	0	33	23.75	0	0
Utilization of beds (%)	99	91	88	74	57	41
Minimum average waiting time (hour)	410.22	14.56	0	0	0	0

Table V shows that patients are subjected to prolonged waiting times, particularly in the pediatric ICU, under Scenario 1. In this scenario, the pediatric and reanimation ICUs are operating at bed utilization rates of 99 % and 90 %, respectively. In contrast, in Scenario 2, the bed utilization rates decrease to 88 % and 74 % in the pediatric and reanimation ICUs, respectively. As a result, in Scenario 2, 33 % and 23.75 % of patients were redirected to alternative healthcare facilities in the pediatric and reanimation ICUs, respectively.

Scenario 3 is a simulation-based approach that aims to ascertain the number of beds that can adequately serve all patients without incurring any wait time. In the event that the pediatric ICU comprises 30 beds and the reanimation ICU has 19 beds, it would be feasible to provide immediate service to all patients without any wait times. In this scenario, the utilization rate of the beds manifests as 57 % and 41 % for pediatric and reanimation ICUs, correspondingly. Notwithstanding the low utilization rate of the beds, it is crucial to bear in mind that ICUs cater to patients in critical circumstances, necessitating prompt medical intervention, and must therefore always have a readily available bed.

5. CONCLUSION

This paper presents a simulation-based model that utilizes mixture distribution as input parameters to manage the bed capacity required to serve all patients in two ICUs of the Adana teaching hospital without delays. Specifically, we offer an approach for the optimisation of the number of beds in ICUs, in order to provide immediate assistance to patients. In practice, it is highly important to have a thorough understanding of the appropriate bed capacity required for ICUs that do not permit waiting. This simulation-based approach enables decision makers to assess the benefit of providing extra beds to minimize the patient rejection rate. However, significant challenges arose due to the highly skewed nature of the *IAT* and *ST* data. To address this issue, mixture distributions were employed to establish *IAT* and *ST* parameters, which were integrated as inputs into the simulation-based model.

The main contribution of this study is its use of mixture distributions to generate input parameters for a simulation model that is then used for capacity planning. This study represents one of the first investigations to utilize a blend of a mixture distribution and a simulation-based approach to determine the bed capacity for ICUs. The advantage of implementing a mixture distribution lies in its consideration of bed resource utilization by both short- and long-stay patients. The significance of longer stays can be attributed to their impact on the availability of beds, as such patients tend to occupy them for extended periods.

The methodology is intended to prove the advantage of the mixture distribution approach in simulation-based models. Nonetheless, in real-world settings, the hospital environment is subject to daily and even minute-by-minute changes. For instance, in this study, the developed simulation model did not take into consideration emergency patients or staffing levels, which could alter the system's dynamics. Thus, future studies could design a system that considers not only the number of beds but also the number of medical staff and equipment, and/or the prioritisation of patients, which could be re-optimized using this approximation.

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