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METAHEURISTIC OF ARBITRARY LUMINOUS INTENSITY DISTRIBUTION FOR ROADWAY LIGHTING LUMINAIRES

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Abstract
The introduction of light-emitting diodes (LEDs) for roadway lighting necessitated a completely new approach in the optical design of lighting systems. The small Light Emitting Surface (LES) of the Solid-State Lighting (SSL) sources enables precise and robust optical control. While for illuminance-based lighting scenarios, the desired light pattern is deduced through the inverse-square law, for the perceived luminance-based calculations, the combined effects of multiple light points make the problem statement for the design objective definition challenging. This research paper presents an original theoretical and practical method for generating a luminous intensity distribution for fulfilling the lighting requirements of luminance-based lighting classes. The results showed that designing optical surfaces to achieve this generated luminous intensity distribution is leading to better task efficiency (often over 10 % power saving potential at same wattage compared to Taguchi method) in shorter computation times. Therefore, the method is desirable for roadway lighting optical design and has a prospect in gaining a better understanding of night-time roadway safety.
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Key Words: Lighting, Outdoor, Roadway, Optics, LED, Evolutionary Algorithm

1. INTRODUCTION

1.1 Motivation

With the introduction of highly efficient LED based street lighting luminaires, a revolution has started in the outdoor lighting market, chasing efficacy, substituting traditional luminaires equipped with discharge light sources, reaching lower power consumption and significantly lower maintenance costs [1]. A third aspect of the new technology was often overseen, which is the higher degree of achievable optical control with modern optics [2]. Mainly due to the small LES of the white light emitting diodes, sharper cut-offs and narrower luminous intensity peaks can be designed with both refractive and reflective optics. This enables higher task efficiency, meaning that the light is driven more precisely to where it is needed [3].

Roadway lighting has always meant a great challenge in the industry, since for these applications, the lighting scene is to be optimized for the perceived luminance by observers (drivers of the motorized traffic). That is – as defined in the governing standards (European Standard Series EN 13201 [4] and ANSI/IES RP-8-18 [5]) – a function of the spatial luminous intensity distribution of the light points, the reflectivity characteristics of the asphalt and the geometrical layout of the lighting scenario. For the lighting designers of streetlights, usually a luminous intensity distribution characterization protocol of a luminaire is given as an input (usually IES [6] or EULUMDAT / LDT file format) and the layout of the scenario. A street lighting luminaire is usually considered better optically from a lighting designer's point of view, if the requirements of the standards for dimensioning the lighting can be met at lower power consumption [7-9]. Other design parameters include the amount of disability glare, uniformity of luminance, Edge Illuminance Ratio (*EIR*) or Surround Ratio (*SR*), illumination of the close environment, and upward light output ratio (*ULOR*) for a given lighting setup [10]. Two less obvious aspects are the optical efficiency and the robustness of an optical system [11] – that are both much rather an optical designer point of interest as these are usually excluded from a products datasheet.

In the optical engineering practice, the typical design workflow is based on the technical evaluation of Monte Carlo type nonsequential ray tracing simulation of the luminous intensity distribution, while optimizing optically refractive and/or reflective surfaces. Such simulations usually take three to five minutes for indicative, quick results and up to an hour for high fidelity approximations with characterized light source, colour information, stray light analysis and complex scattering models. Experienced optical designers may control over 50 design variables [12] for a roadway lighting primary optics design with parametrized optical surfaces. These each can influence the resulting luminous intensity distribution to some extent and are rarely independent. This concludes that engineers cannot optimize one variable at a time, thus gradient methods – even such as Taguchi design optimization method – rarely lead to strong designs and the design space is practically unmappable [13]. (Designs are iterated to a point that seems acceptable technically, but is not proven to be an extremum in any aspect for the overall parameter space.) Additionally, this method inherently focuses the optimization on real, practical task efficiency, that is complicated to control with traditional methods and is the most essential aspect of a financially feasible public lighting [14, 15].

1.2 Methodology

A proposed method by the authors to overcome these obstacles was to optimize the design to minimize the mean squared error (*MSE*) to an existing luminous intensity distribution that is previously identified as a good fit for a lighting scenario. The advantage of this is twofold. Once, the merit function for the design study only focuses on a single numeric value instead of maximizing perceived luminance, while reaching certain uniformity, controlling threshold increment, and so on. Secondary, this method is less sensitive to the fluctuation observed in those cases over the design space, when while designing for the reflected light from the tarmac, minor changes in a specific spatial luminous intensity value can have enormous, non-linear impact on the results [16, 17] – often at a higher resolution than the optimization steps [18].

Multi-Objective Genetic algorithms usually perform outstanding, approximating solutions to all types of problems with multiple variants of significant, non-linear influence to the merit function because they ideally do not make any assumption about the underlying fitness landscape [19]. It makes this approach suitable for the problem statement of luminance-based roadway lighting optimization, as there is no canonical relationship between visual performance and the extent of local luminous intensity values. The optimization was enhanced with a Hybrid Code Genetic Algorithm, as it was proven useful by dynamic mutations in avoiding early traps and also accelerating convergence [20].

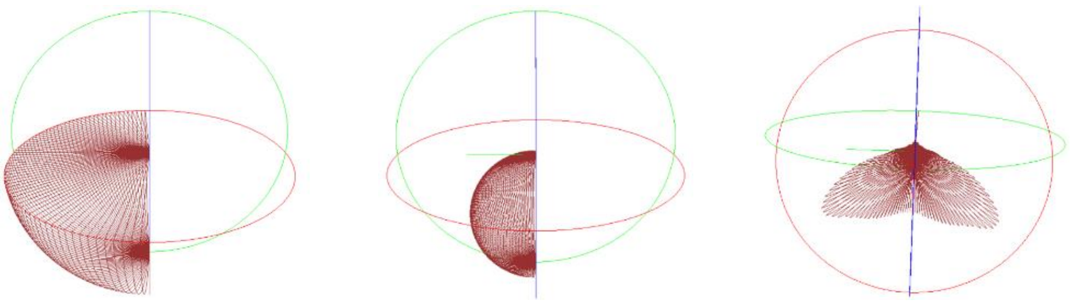
2. INTRODUCTION OF THE ALGORYTHM AND METHODS USED

2.1 The workflow of the proposed algorithm

For the algorithm, a general file format and data format was used that is well known in the lighting industry and is commonly referred to as “IES File”; specified in the IESNA:LM-63-2002 Standard. This data structure contains the luminous intensity values in candelas for three-dimensional space, arranged in a specified spherical coordinate system. The array containing the luminous intensity magnitudes in spatial directions is referred to as I-Table. Both the zenith and the azimuth resolutions are defined in a two-dimensional array in the header of the data structure. For the scope of this paper, a homogenous partitioning was used with varied resolutions. Only the bottom 90-degree part of the polar angles was used, permitting no *ULOR*, and only one half of the azimuthal hemispace, considering only that part of the luminous intensity distribution which is directed towards the roadway. Luminaire tilting is also out of scope of this paper. As for the last practical limitation, it was also ensured that the intensity distributions were symmetrical to the plane perpendicular to the roadway lanes. These restrictions can be suppressed and are only implemented for better understandability in this research paper.

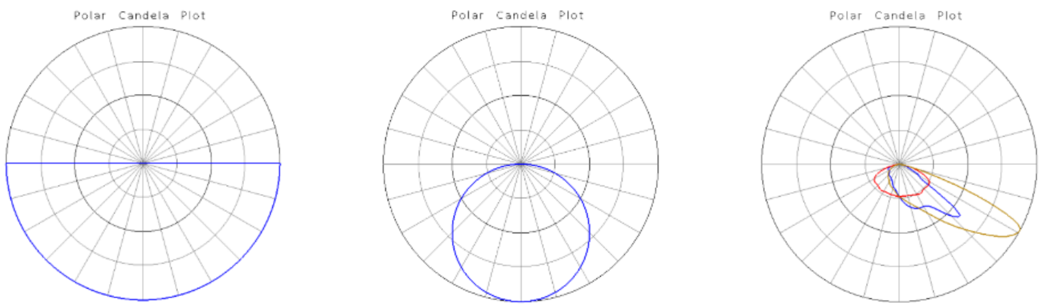
2.2 Baseline luminous intensity distributions

The initial luminous intensity distribution that is optimized with the developed algorithm is called a baseline. There were three such baseline datasets used; one containing equal luminous intensity values in all relevant directions so that the luminous flux was normalized to 1000 lm, later referred to as "iso-candela" baseline. The second was established for quicker gains of results, determining the intensity values in a way, that the cosine of the polar angle (from pointing downwards) was proportional to the intensity and the luminous flux was again 1000 lm. The third baseline was the goniophotometer measurement of a commercially available LED based streetlighting luminaire, where only the relevant directions have been used and all other values have been cropped. This latter one was also normalized to a luminous flux of 1000 lm. In this initial setup, the algorithm creates an array of a defined size of the baseline intensity distributions. Figs. 1 and 2 are showing the intensity diagrams of each of the three baselines in polar diagrams and orthogonal projection 3D representations. The main intent of introducing these different initial sets is seemingly important for the investigation of the robustness of the method, as each iteration leads to practically the same result with the later introduced merit functions, no matter what the baseline is. This test has been performed several times during the development of the method and it was found that evolutionary algorithm itself does not guarantee robustness with any continuous and monotone evaluation function.



a) b) c)

Figure 1: Projective representation of the: a) isocandela baseline, b) spherical baseline, c) intensity distribution of the commercially available product.



a) b) c)

Figure 2: Polar plot representation of the: a) isocandela baseline, b) spherical baseline, c) intensity distribution of the commercially available product.

2.3 The merit function

The developed algorithm calculates the lighting parameters of the baseline luminous intensity distribution to a pre-set metric in a given lighting scenario. It was prepared to be able to conduct the full calculation routine for EN 13201-3:2015 and ANSI/IES RP-8-18, Luminance Method and Small Target Visibility Method (including custom parameters and custom obstacles [21-23]). All partial results are stored in the program memory and can be the variables for a customizable merit function, testing the underlying results in each step of the iteration.

2.4 The mutator class for inducing slight deviations

In the next step of the process, various deviations of the baseline or preceding elements of the iteration are created. Just like the merit function, this mutator method can be customized. The algorithm must be configured for the number of deviated offspring to be created (mutants) and for the type of the mutations. Methods used for this paper were:

- Scaling a random (or systematically each) intensity value by a random extent between specified limits (e.g. 80 % to 120 %).
- Scaling multiple random intensity values by a random extent between specified limits.
- Scaling a random intensity value by a random extent between specified limits and scaling the surrounding intensity values in a circular pattern with a specified decay linearly.

For the third method, the dampening of the deviation was done according to Eq. (1):

$$m = \frac{d}{\sqrt{(\Delta\theta)^2 + (\Delta\varphi)^2}} \tag{1}$$

where m is the dampening of the deviation multiplier, $\Delta\theta$ is the difference in the azimuthal angle from the central intensity value, $\Delta\varphi$ is the difference in the polar angle from the central intensity value and d is the weighting parameter for the algorithm. Parameter d was set to 1 in the scope of this paper for simplification. Higher values of parameter d may lead to quicker increase of the merit function but also make the algorithm unstable (not converging). In the next step, the merit function was recalculated in each instance of the iteration with all newly begotten intensity distributions. While there are many strategies with genetic algorithm from this point to proceed, only those intensity datasets with the best merit functions were stored and got forwarded for the next iteration process. The number of solutions kept could also been configured. Principal Component Analysis techniques [24] in the generator process have shown significant increase in the agility of this method. Multi-objective optimization and the identification method described by Choi & Kim [25] has also a great potential, according to early tests, but these are not in the scope of this paper.

This process then got repeated a configured number of times (iteration) or until the change in the merit function was less for a set number of iterations than a set proportion. This algorithm is based on the fact that every time a better fitting solution is found (at random) than the previous known best, this new instance will be more optimal than the previous for the specified problem statement (expressed with the merit function).

The reason for not only the very best merit function is kept for future consideration roots in the observation that sometimes this algorithm may find local limits for the optimization. While there are more advanced methods to increase the performance of the genetic algorithm, this described process was used for the current scope with adequate results.

It is to be noted that in case the number of deviations is set to double the number of relevant intensity values of the intensity distribution dataset, each instance is evaluated, increasing and decreasing its numeric value and examining its effect on the merit function. During our investigations, randomly scaling the individual intensities always led to the same result as the method considering all possible options (above a lower limit of mutation operators, as will be shown later), with a population of 1/30th of the complete set. We considered two results the same, when there was no more than 1 % absolute difference in any specific intensity value over two entire datasets. This is also true for the various baselines, meaning that the algorithm has concluded the same result, no matter what the baseline was.

For a faster convergence of the results, a dampening on the deviation extent can also be configured with a constant. This is a simple multiplier to the upper and lower specification limits with each iteration, as described by Eq. (2):

$$mD = 1 \pm (l_d * d_{damp}{}^{i}) \tag{2}$$

where D is the multiplier to the randomly selected intensity value, l_d is the set deviation limit (e.g. 20 %), d_{damp} is the pre-set dampening on the deviation and i is the iteration number.

2.5 Termination conditions

The iterative optimization runs until a termination condition is reached. Since the optimization is based on random distortion of an input dataset, it can't be proven that a result obtained will be a global solution to the problem. Test runs have shown that a low population leads to unstable results. In this case, it is possible that for multiple runs, the algorithm deviates the merit function towards a less efficient solution and might even lead to non-convergence.

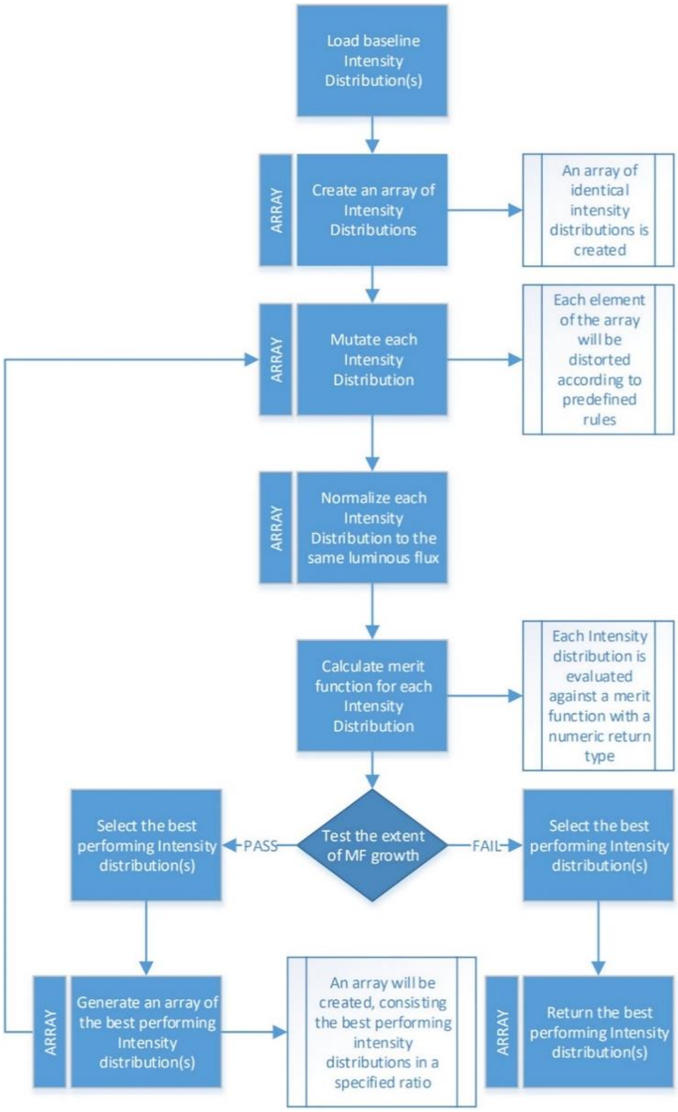


Figure 3: The workflow outline of the algorithm.

Fig. 3 shows the workflow of the metaheuristic used, from initiation to the termination condition and process return. The test method for the adequacy of the resulting set after a given number of iterations is applied to every instance of the current generation. The evaluation can be configured to trigger pass as long as there is a reason to further optimize the population which depends on the characteristics of the dataset and mutator function. In this specific case, acceptance criteria depend on the count of the population, as lower densities lead to higher chance of no improvement in a given generation.

3. ANALYSIS AND INTERPRETATION OF DATA

3.1 Tuning the variables of the algorithm

Fig. 4 shows a brief evaluation of these investigations in terms of population setup. For the purpose of testing the effectivity and consistency of the algorithm, multiple setups have been run multiple times. The reference setup was a full-scale optimization, where each instance in the intensity distribution dataset was evaluated in each iteration. The baseline luminous intensity distribution was an iso-candela type, 2-degree azimuth and 1-degree nadir spatial resolution, evaluated only in directions, where there was an intersection with the roadway surface. This meant an overall of 5670 degrees of freedom (*DoF*). The lighting scenario in this presented case was a 5 m wide, 2 lane roadways with a light point at 10 m mounting height and 25 m pole inter-distance, one-line distribution, no tilting. Each case was run for 1000 iterations before a programmed termination (no evaluation of merit function change in this case).

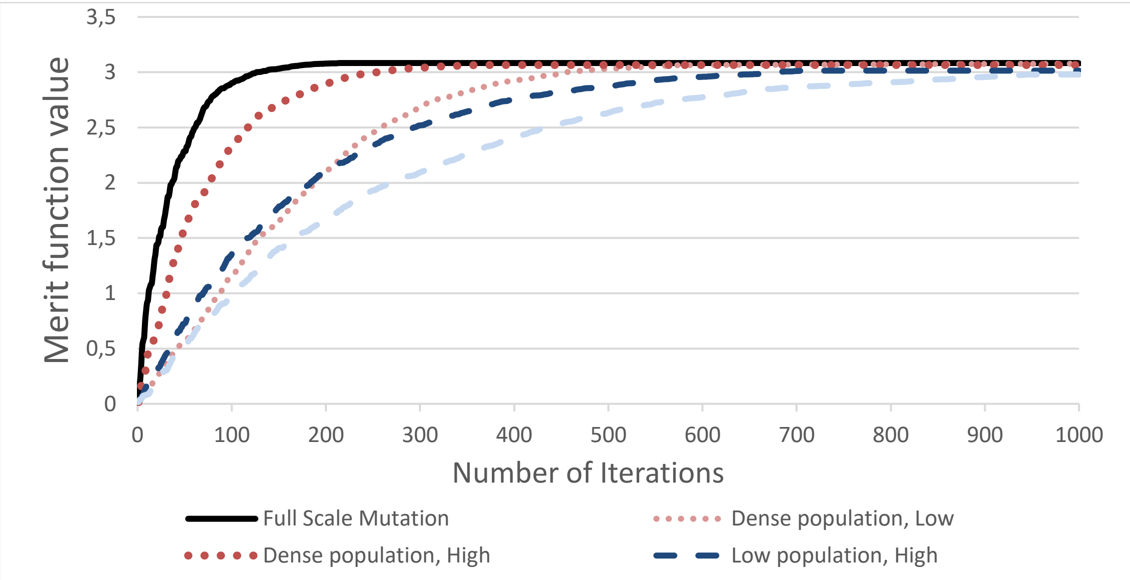


Figure 4: Comparison of algorithm responses with different setups.

The process with the reference setup took a bit over 1 hour runtime on a high-performance desktop PC (48.3 Giga Floating Point Operations per Second – GFLOPS), reaching the 95 % of the peak merit function value after one hour. It was done 3 times on 3 different days and gave the same numeric results on each optimization (as expected, since there was no randomness in this setup).

For the setup with ‘Dense population’, a population of 189 was used. The ‘Low population’ setup performed only 20 mutations in each iteration at random. Both setups were executed 10 times, in alternating order. The chart in Fig. 5 indicates the merit functions value at each iteration for the three different setups, showing the fastest and slowest reach of 95 % of the merit function in each case.

Fig. 5 shows on a similar chart an average of 3 iterations in each case, running the same optimization on the lighting scenario with the three different baseline sets. The commercial product initiates at a much higher value of a merit function. Eventually, each setup reaches to the same resulting luminous intensity distribution. The initial slope of the non-optimal baselines is steeper as a change in a given directional luminous intensity contributes more than in the case of an already fitting setup.

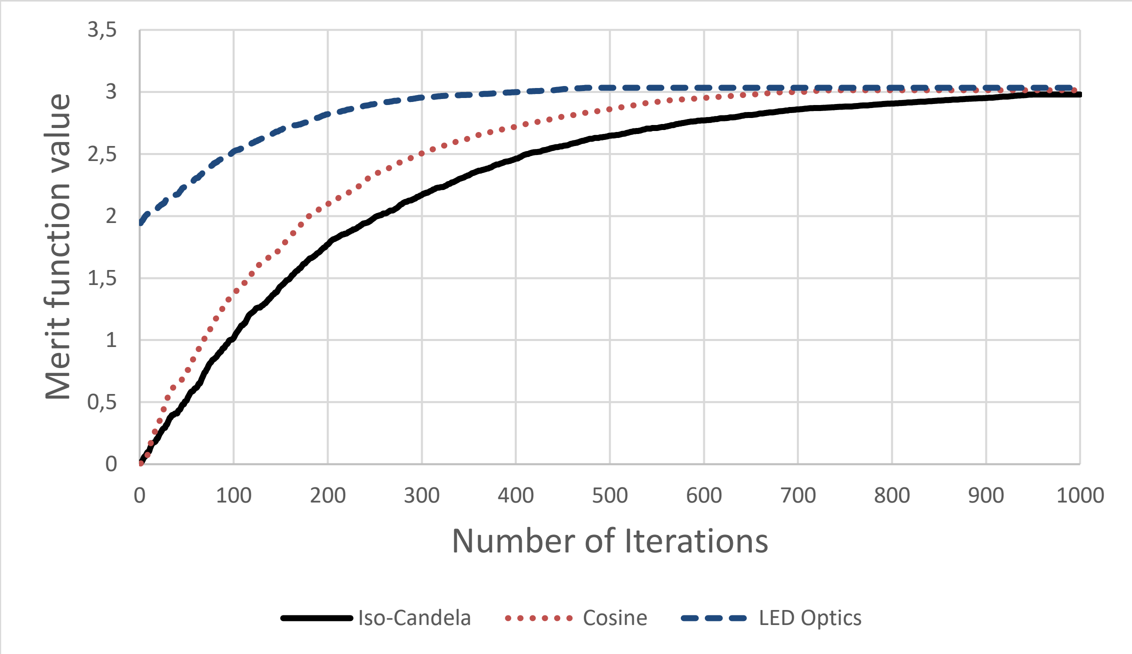


Figure 5: A comparison of algorithm performance with different baselines.

3.2 The evaluation of the luminous intensity distributions by merit function

The introduced algorithm does nothing else, but finds the luminous intensity value change, that has the most positive impact on the lighting design is a specified scenario and repeats this until the result cannot be further improved.

The merit function used determines if the method is effective or not. For the variables of these fitness functions, a parameter set is calculated over the defined roadway lighting scenario. The optimization routine was designed to converge to the maximum of Eq. (3):

$$MF = f_{Lav}(L_{av}) * f_U(U0) * f_U(Ul) * f_{TI}(TI) * f_{SR}(SR) * f_{LI}(LI) \tag{3}$$

where *MF* is the merit numeric value of the Merit Function and the variables used are calculated as defined by the EN 13201 standards:

L_{av}: Average Luminance, cd/m²

*U*0: Overall uniformity

Ul: Longitudinal uniformity

TI: Threshold increment

SR: Surround ratio / *EIR*: Edge Illuminance Ratio

LI: Luminous Intensity Class handled as an integer (e.g. G3 is 3)

Figs. 6 to 10 are introducing transfer functions used of Luminance, Uniformity, Threshold Increment, *SR* and Luminous Intensity Classification used by the merit function for the evaluation of any given member of the arbitrary population. The resulting luminous intensity distribution is bilaterally symmetrical and using high-resolution array results spikes towards the calculation grid points. These fitness function can be numerically weighted by applying a power function to them.

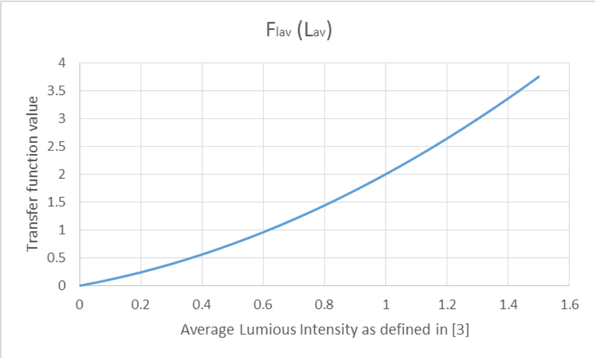


Figure 6: Transfer function of average luminance.

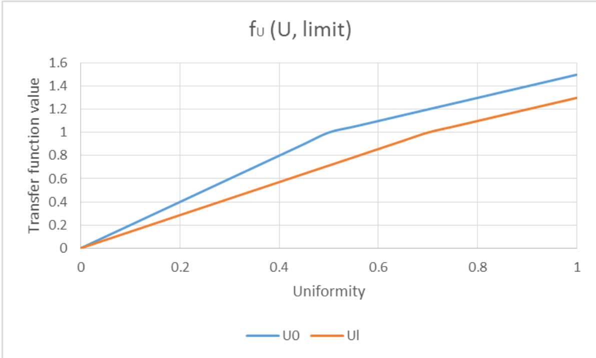


Figure 7: Transfer functions for EN 13201 uniformity metrics.

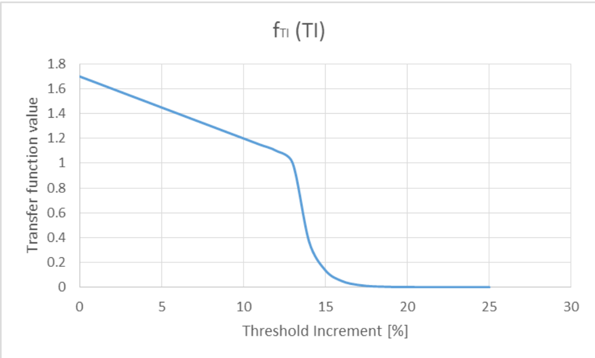


Figure 8: Transfer function for Threshold Increment (TI).

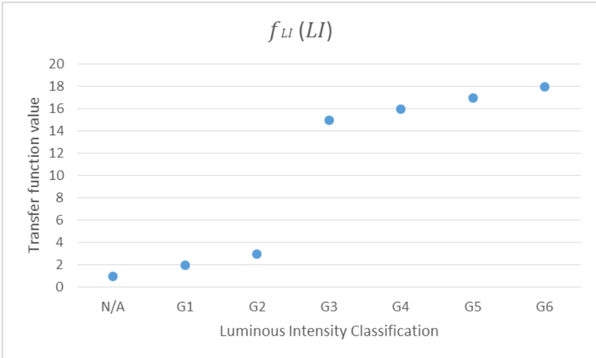


Figure 9: Transfer function for luminous intensity classification.

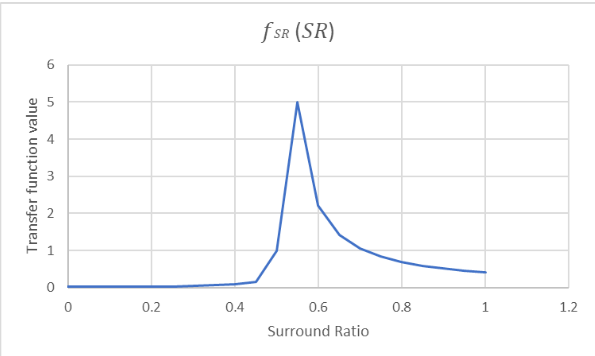
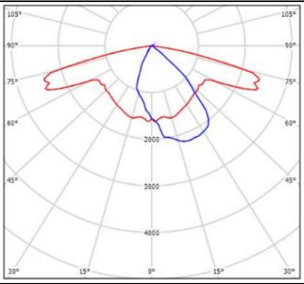
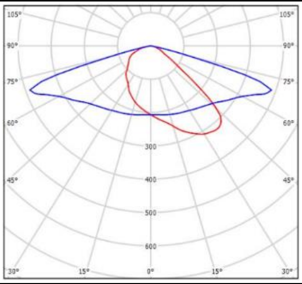
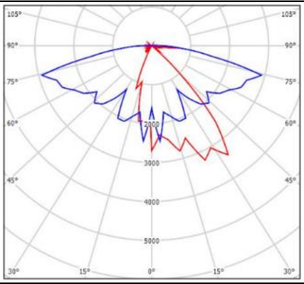


Figure 10: Transfer function for surround ratio.

The need for luminous intensity classification was implemented in order to maintain integrity of similarity by the high angle luminous intensity outputs of the generated distributions. Then, using the ratios indicated in Fig. 9, the undesired luminance accumulation above the 70-degree Nadir angle caused by highly specular reflective roadway surface can be dampened. Even though the standard recommends calculating this only in case if *TI* is not applicable, there are increasing number of high-volume installations with LED light source, where a limitation of this rating is also demanded by municipalities and lighting design validation [26-28].

In order to benchmark the capabilities of the algorithm, several trial roadway lighting designs were program-automatically created to compare the performance with two commercial products catalogue luminous distribution at the same luminous flux, that are meeting every visual requirement of these scenes. As an example, with single row layout, 32 m pole distance, 10 m mounting height, no overhang and 8 m width roadway with R3 type tarmac, the results turned as shown in Table I.

Table I: Comparing evaluation of capability on a lighting scenario.

		Product A	Product B	Generated Luminous Intensity Distribution
	Requirement			
L_{av} [cd/m^2]	≥ 0.75	0.94	0.75	1.26
*U*0	≥ 0.40	0.44	0.49	0.6
Ul	≥ 0.60	0.61	0.66	0.92
TI [%]	≤ 15	10	6	14
SR	≥ 0.50	0.55	0.55	0.52

4. SCIENTIFIC AND INDUSTRIAL APPLICATIONS

4.1 Practical application within optical design engineering

As a practical example, it was tested if there is any gain in the automated processing time for the optical surface design of a street lighting LED luminaire reflector, comparing merit functions on a two variable optimization scenario. In the reference case, a Design of Experiments (DoE – an embedded feature of the optical design software based on Taguchi-method) based algorithm was used to optimize the uniformity, threshold increment and task efficiency (based on European standard EN 13201). In the proposed method, the same algorithm did the same parameter optimization for minimizing the mean average difference between the simulated intensity distribution and an arbitrary one, generated with the introduced metaheuristic. The optical source was the same in each case. The advantage in a practical application comes from the fact that tuning the variables of the optical surface for improving visual performance is chaotic; with the algorithm likely finding local extrema. Tuning a surface on the other hand to match a reference output is robust in aspects of changing variables are either increasing or decreasing the cumulative residual (difference) between the two.

Table II shows the visual performance of the intensity distributions simulated with the resulting optical surfaces after the two approaches.

Table II: Comparison of the optimization capabilities over the input lighting scenario.

	DoE around lighting design	Proposed algorithm
Average Luminance [cd/m^2]	0.56	2.05
Uniformity *U*0	0.63	0.78
Threshold Increment [%]	8	10

The design optimization was run 16 times for both cases and each study contained 25 iterations, running for about 5 minutes each time. Number of rays traced was 5 million in every

Monte Carlo simulation, up to 8 subdivisions splitting on surfaces was applied, no colorimetric analysis included with 4000 K colour temperature LED spectrum model with the model meshed to a minimum resolution of 20 μm. The proposed algorithm results an acceptable design output after this process. The DoE method however was still investigating a local extreme in the case of the optimization based on the lighting design in the 16th iteration.

Continuing the optimization from this point lead to the results summarized in Table III. The built-in method in the optical design software returned a local maximum with a passing result for the requirements of the lighting scenario. Reaching this point needed 42 sets of simulations total in this specific case. With the proposed approach, the better performing result was reached in only 20 runs.

Table III: Comparison of the capabilities over the input lighting scenario after optimization end condition.

	DoE method	Proposed algorithm
Average Luminance [cd/m^2]	1.48	2.25
Uniformity *U*0	0.69	0.84
Threshold increment [%]	12	10

5. CONCLUSION

A novel algorithm was shown in this paper for creating arbitrary luminous intensity distributions for roadway lighting applications for multiple standards. The proposed method is based on genetic algorithm for finding a well-fitting solution to an optimization problem with thousands of variants and complicated merit functions. The method was shown to be useful for optical design procedures of roadway lighting luminaires. It is possible to highly automate design processes with the method presented. Using the introduced practices, it is possible to design customized optics for specific lighting scenarios with a high level of automation and excellent task efficiency.

The method can also be used for the experimental evaluation of different roadway lighting standards, cross validation of product concepts and for gaining a better understanding of task efficiency [29-31].

The introduced method generates an outstanding solution to a given lighting problem statement; however, it is very specific and thus impractical to have a fine-tuned product that might not perform well in a slightly different lighting task. Practical evaluation of cases highlighted the need for the implementation of simultaneous optimization for parameter intervals, multiple R-Tables, considering the fact that the reflectivity characteristics of the roadway surfaces change during the lifecycle of a roadway and this can lead to major impacts in the results of this method.

The findings described in this paper may provide a novel input in the understanding of disability glare in roadway lighting applications and advanced perception models could be implemented in the future for evaluating current practices in modelling veiling luminance, such as described in various papers [32, 33].

A novel practical design method was shown for roadway lighting optical design. This new proposed workflow might be able to provide a higher level of user focus by implementing a design review of the arbitrary intensity distribution generated with the metaheuristic before performing the optical surface design.

A working online application with full functionality addressed in this paper is available at the referred web address: *https://www.meerkatblog.com/roadwaybf* [34].

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