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HOW DISCRETIZATION AFFECTS INTERMITTENT DEMAND STOCK MANAGEMENT BASED ON SIMULATION

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Abstract
This paper is aimed at the development of an alternative combinatorial strategy of reducing searched solution space in intermittent demand stock management based on the past stock movement simulation. The combinatorial strategy involves an adjustable level of the discretization of control variables that are used within a selected inventory control policy. We combine this new strategy with the local search employing linear regression and bootstrapping to bound the reorder point and simulate (Q, R) inventory control policy using randomly generated data. The data is characteristic with an increasing intermittency and a non-zero demand variability. The outputs from simulation experiments show that combining two different strategies of reducing searched solution space brings a significant improvement in the trade-off among the minimal holding and ordering costs, required service level and the consumption of the computational time making the past stock movement simulation to be more applicable in extensive real life tasks.
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Key Words: Logistics, Intermittent Demand, Stock Management, Simulation, Optimization

1. INTRODUCTION

Many manufacturing companies consider the management of their inventory of spare parts to be a crucial task [1]. From the maintenance point of view spare parts represent a material input ensuring a production equipment to keep on running smoothly [2]. Furthermore, spare parts are also a subject to the after sales activities covering the whole life cycle of final products [3]. Speaking of spare parts inventory, there are capital-intensive industries (i.e. chemical, car manufacturing or aerospace) in which to come out of stock is considerably detrimental, both from financial and operational perspective [4].

The logistics of spare parts is highly complex mainly because of the demand characteristics encompassing a low demand frequency and high variability in the demanded quantities [5]. Demand sporadicity and irregularity make especially inventory control of spare parts a very challenging task. When employed in an estimating the demand during an order lead time period, traditional parametric methods such as exponential smoothing (SES) do not perform well [6]. That is why Croston modified SES incorporating the frequency of occurrence of non-zero demands in estimating mean and variance of lead time demand and laid the foundation for more efficient forecasting and stock control. Throughout the years, many researches were interested in improving the performance of Croston's method (CR). See for instance some interesting suggestions presented in [7-10]. Croston's approach has been also incorporated to the various software for the forecasting and stock management of sporadic demand usually adapting a demand classification scheme considering an erraticity and an intermittency of the demand to select the most suitable modification of Croston‘s approach for a specific demand type (see e.g. [11-13]). Because parametric methods such as SES or CR estimates an average demand per period and subsequently this estimation becomes an input to the calculation of the demand during order lead time there are several drawbacks that one has to be aware of when using this kind of approach in inventory control. Firstly, smoothing constants have to be optimized requiring to choose a forecast accuracy measure. However, for a selected accuracy metric, a different parametric method can emerge as the most suitable for inventory control of an item with a specific demand behaviour [14]. Secondly, some assumptions on a standard demand distribution are adopted potentially influencing the estimation of the lead time demand in a negative way [15].

Nonparametric methods, on the contrary, do not make this kind of assumption and therefore they can be used successfully in situations when the lead time demand is hard to describe with help of a theoretical distribution [16]. These methods involve especially bootstrapping, so called empirical method, a neural network application and the past stock movement simulation.

Classical bootstrapping introduced by Efron [17] and modified for intermittent demand applications by Willemain et al. [18] uses a sampling from a demand data coming from the past and empirically generates a lead time demand distribution. Based on a scientific study using a real data coming from several different industries, Willemain et al. show that their modification of original bootstrapping method outperforms single exponential smoothing as well as Croston’s method. As in case of Croston’s method several suggestions how to increase the efficiency of bootstrapping can be found in scientific literature (see e.g. [19, 20]). Nevertheless, a key issue with bootstrapping method is that demand irregularity can cause considerable overestimation of lead time demand resulting in a poor performance whether applied in intermittent demand inventory management [21].

So called empirical method [22] avoids the sampling which is the essential component of bootstrapping and facilitates fast generation of a lead time demand distribution. When tested on a real data coming from crude oil processing the empirical method performed better than bootstrapping but worse than a parametric method based on normally distributed demand. The use of extreme value theory to represent the tail of the lead time demand distribution [23] and the incorporation of randomness into lead times [24] are two intriguing adaptations and extensions of the empirical method.

Currently, applications of neural networks represent another growing scientific stream in sporadic demand forecasting and inventory control. Neural networks as an artificial intelligence method are broadly applied in the industrial sector to solve diverse tasks [25]. The advantage of this method is an ability to recognize a demand behaviour right from the data and capture intermittency and irregularity more precisely than other techniques [26]. For readers that are further interested in this scientific stream we recommend discovering the work of Guo et al. [27] or Shafi et al. [28].

And finally, another nonparametric approach represents the past stock movement simulation (PSMS) proposed by Dyntar and Kemrova [29]. In PSMS a simulated period is separated into time intervals of the same length and with a certain demanded quantity that comes from the past or that can be generated. For each interval there are three possible and subsequent actions to be simulated involving a replenishment, a demand satisfaction directly from available inventory and an ordering. The ordering is controlled by a selected inventory policy employing continuous or periodic review and constant or variable order quantity. The advantage of the simulation is that the in-depth discretization of time and the repetitive run of the simulation for a different combinations of controlled variables certainly brings better performance in term of holding and ordering costs than other parametric and nonparametric techniques. Unfortunately, if total demand is too high, the solution space becomes very extensive and requires huge amount of computational time to be searched that almost excludes the method from possible applications in real life tasks.

In this paper we examine how a different level of discretization of reorder point (R) and replenishment order quantity (Q) affects both the best reached holding and ordering costs and the consumption of the computational time in PSMS of continuous review, fixed order quantity inventory control policy [i.e. (Q, R)]. Thus, we gradually set the level of discretization of Q, R from 2 (i.e. $\mathrm{AC}_{\mathrm{step}_2}$) to 5 (i.e. $\mathrm{AC}_{\mathrm{step}_5}$) pieces expecting to reduce a solution space that needs to be searched when compared to original all combinations search proposed by Dyntar and Kemrova (i.e. $\mathrm{AC}_{\mathrm{step}_1}$). More specifically, if total demand in a time series for an item is equal for example to 5 pieces the simulation with $\mathrm{AC}_{\mathrm{step}_1}$ requires checking 10 combinations of *Q-R* (i.e. 5-4; 5-3; 4-3; 5-2; 4-2; 3-2; 5-1; 4-1; 3-1; 2-1) while $\mathrm{AC}_{\mathrm{step}_2}$ reduces this number to 3 *Q-R* combinations (i.e. 4-3; 4-1; 2-1) assuming that only $Q > R$ combinations are permitted. We test this sort of a combinatorial nature optimization approach in conjunction with our paper previously published in the *International Journal of Simulation Modelling* (see [30]) that uses a different strategy to reduce the searched solution space. This strategy entitled the local search (LS) involves underestimating *R* with the help of linear regression and to overestimate *R* with bootstrapping vice versa. Then the simulation runs with the reduced *R* interval as $\mathrm{AC}_{\mathrm{step}_1}$ Whether we go back to our example of the item with the total demand equal to 5 pieces assuming that linear regression for the time series returns for example $R = 1$ and bootstrapping for example $R = 2$ the number of simulated *Q-R* combinations is 7 (i.e. 5-2; 4-2; 3-2; 5-1; 4-1; 3-1; 2-1). When simulating randomly generated demand data with 20-70 % zero demand periods and with demanded quantities per period ranging from 1 to 25, 1 to 50, 1 to 75 and 1 to 100 pieces, the results published in [30] show that LS brings substantial reduction of computational time compared to $\mathrm{AC}_{\mathrm{step}_1}$. On the other hand, the best possible holding and ordering costs are reached just for up to 50 % of simulated time series and although for another 40 % of simulated time series the maximal difference in these costs is up to 15 % there is still a significant potential for an improvement. Thus, in this study we are interested in answering following research questions:

- Does a different strategy of reducing the searched solution space (i.e.$\mathrm{AC}_{\mathrm{step}_{2-5}}$) outperform LS in term of the consumption of computational time and the best reached holding and ordering costs?
- Does the combination of LS and $\mathrm{AC}_{\mathrm{step}_{2-5}}$ bring improvements in term of reaching/getting closer to the best possible holding and ordering costs reached by $\mathrm{AC}_{\mathrm{step}_1}$ while maintaining the advantage of significantly lower consumption of computational time?

We further organize this paper as follows: In Section 2 we describe PSMS and the arrangement of simulation experiments. We also summarize basic features of simulated demand data coming from [30] including an elimination of some simulated scenarios. Section 3 contains conclusions and the contribution of authors to the development of the field of study.

2. METHODOLOGY AND DATA

2.1 Past stock movement simulation

Initially, we use PSMS of (*Q*, *R*) inventory policy described in [30], Appendix 1. We remove the calculation of underestimated reorder point with help of regression as well as the sampling used in the maximal reorder point calculation with help of bootstrapping. Then we add in For/Next cycles to combine *Q*, *R* and we also extended these cycles by the option to adjust the step size. We use a required fill rate (*FR*) as the service level indicator describing the ability to satisfy the demand right from the available inventory [31-34]. Similarly to [30], in the simulation, we continue to avoid back orders in case of insufficient inventory and multiple orders in the pipeline for an item. On the contrary, we permit a partial demand satisfaction causing a relevant missing quantity becomes the part of an achieved fill rate calculation (*AFR*).

For a simulated combination of *Q*, *R* with $AFR \geq FR$, the best reached total holding and ordering costs (C_t) are for an item calculated as:

$$C_t = AI \cdot T \cdot p \cdot c_h + N_o \cdot c_o \tag{1}$$

where *AI* means an item average inventory, *T* is the duration of simulation, *p* represents an item price, c_h holding costs common to all items, N_o the total number of placed orders for an item and finally c_o represents ordering costs common to all items. As we do not want to commit a stock out in the very beginning of the simulation run we set the initial inventory (*II)* as:

$$II = \sum_{t=1}^{LT} S_t \tag{2}$$

where *t* represents a period, *LT* an order lead time and S_t is a demand per period. To calculate the consumption of the computational time in the simulation, initial and end times of a simulation of a scenario are recorded using predefined MS Excel function *NOW*().

2.2 Simulation experiments and data

To do simulation experiments we use the same data set as in [30]. This original data consists of fifteen scenarios, in each scenario there are ten thousand time series each covering fifty time periods. Demand in a period is generated in a two stage process including a random generation of non-zero demands ranging uniformly from a minimal value (i.e. $S_{t,min}$) to a maximal value (i.e. $S_{t,max}$) and a subsequent replacement of randomly chosen non-zero demand periods with zeros according to a required level of intermittency. For more detailed description of the demand data generation please see section 2.2 in [30] and Appendix A in [5]. Table I then shows demand characteristics of the original data set.

Table I: Basic characteristics of the original demand data set.

Scenario	$S_{t,min}$ [pieces]	$S_{t,max}$ [pieces]	***S*** [pieces]	***ADI***	CV^2	**Time series demand type**
1	1	5	86 – 154	1.25	0.08 – 0.43	Smooth (100 %)
2	1	25	350 – 675	1.25	0.09 – 0.64	Smooth (99 %); Erratic (1 %)
3	1	50	631 - 1 359	1.25	0.09 – 0.64	Smooth (98 %); Erratic (2 %)
4	1	75	966 - 2 020	1.25	0.12 – 0.75	Smooth (98 %); Erratic (2 %)
5	1	100	1 230 - 2 691	1.25	0.11 – 0.75	Smooth (97 %); Erratic (3 %)
6	1	5	47 – 101	2.00	0.05 – 0.51	Intermittent (100 %)
7	1	25	189 – 470	2.00	0.06 – 0.79	Intermittent (97 %); Lumpy (3 %)
8	1	50	351 – 874	2.00	0.07 – 0.84	Intermittent (95 %); Lumpy (5 %)
9	1	75	509 - 1 354	2.00	0.06 – 0.90	Intermittent (94 %); Lumpy (6 %)
10	1	100	694 - 1 768	2.00	0.07 – 0.89	Intermittent (94 %); Lumpy (6 %)
11	1	5	25 – 66	3.33	0.03 – 0.60	Intermittent (100 %)
12	1	25	87 – 290	3.33	0.04 – 0.90	Intermittent (93 %); Lumpy (7 %)
13	1	50	155 – 580	3.33	0.04 – 1.02	Intermittent (91 %); Lumpy (9 %)
14	1	75	262 – 905	3.33	0.05 – 1.33	Intermittent (90 %); Lumpy (10 %)
15	1	100	373 - 1 219	3.33	0.04 – 1.06	Intermittent (90 %); Lumpy (10 %)

In Table I $S_t > 0$ ranges from 1 to 5; 25; 50; 75 or 100 pieces and number of zeros in time series is 20 %; 50 % or 70 %. Total demand for a time series (*S*) is calculated as:

$$S = \sum_{t=1}^{50} S_t \tag{3}$$

To assess a demand sporadicity for a scenario we calculate Average Demand Interval (*ADI*) as [11]:

$$ADI = \frac{50}{\text{Number of non zero demand periods}} \tag{4}$$

To assess a non-zero demand variability for a scenario we calculate Coefficient of Variation (CV^2) as [11]:

$$CV^2 = \left(\frac{\text{Demand standard deviation}}{\text{Average demand}}\right)^2 \tag{5}$$

And finally, to decide on a type of demand for a scenario we employ the classification scheme proposed in [11]. This scheme is shown in Fig. 1.

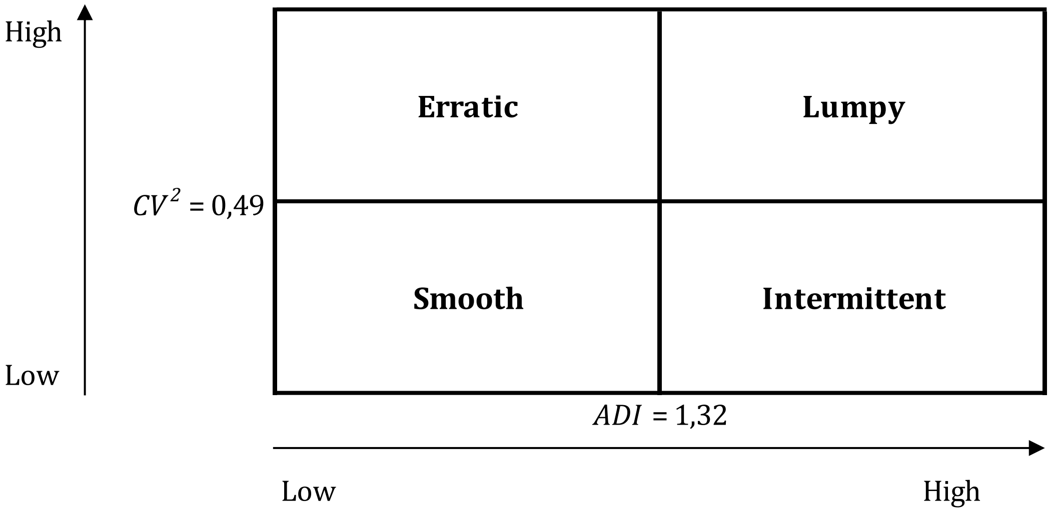

Figure 1: Sporadic demand classification scheme proposed in [11].

Using the results coming from [30] we further eliminate scenarios in which the consumption of computational time for AC_{step_1} ≤ the consumption of computational time for LS (see Table II) as it makes no sense to somehow change the discretization level of control variables in this case. The eliminated scenarios (i.e. 1; 6; 11 and 12) are highlighted with a red font both in Table I and Table II.

Table II: Comparison of computational time of $\mathbf{AC_{step_1}}$ and LS [min] coming from [30].

Signal by / Scenario	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
AC_{step_1}	7	110	397	889	1548	4	45	163	356	622	2	17	59	129	226
LS	31	38	62	100	150	29	37	53	74	115	30	32	41	54	74

The remaining scenarios (i.e. scenarios 2-5; 7-10 and 13-15) become the subject to simulation experiments with the following parameters, given in Table III.

Table III: Parameters of simulation.

p	150	€ / pc
c_h	28 %	from AI in € / period
c_o	35	€ /order
FR	95 %	
LT	3	periods

The remaining scenarios are then simulated in four arrangements differing in a level of the discretization of Q, R ranging from 2 to 5 pieces (i.e. $\mathrm{AC}_{\mathrm{step}_{2-5}}$). All simulation experiments are performed on computer with the processor Intel Core i7 – 2,8 GHz, 16 GB RAM.

2.3 Research results

Using the values of C_t coming from simulation of a scenario (i.e. $C_{t,AC_{\mathrm{step}_{2-5}}}$) and the best possible holding and ordering costs (i.e. $C_{t,AC_{\mathrm{step}_1}}$) coming from [30] a cost difference (Δ) for an item in a simulated arrangement is calculated as:

$$\Delta = \frac{C_{t,AC_{\mathrm{step}_i}} - C_{t,AC_{\mathrm{step}_1}}}{C_{t,AC_{\mathrm{step}_1}}} \cdot 100\ \%\ \text{for}\ i = 2, 3, 4, 5 \quad (6)$$

where i represents the level of the discretization of control variables. Then, for each scenario we use MS Excel function *PERCENTILE*() to calculate 1 to 100 % percentiles of cost differences and we put these percentiles together with LS percentiles coming from [30] in Table IV to make a comparison.

Table IV: Cost differences Δ (part 1 of 2).

	Percentile										
Scenario	**10 %**	**20 %**	**30 %**	**40 %**	**50 %**	**60 %**	**70 %**	**80 %**	**90 %**	**100 %**	***R* by**
2	0 %	0 %	0 %	0 %	0 %	1 %	4 %	8 %	13 %	54 %	LS
	0 %	0 %	0 %	0 %	3 %	5 %	7 %	10 %	13 %	54 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	4 %	6 %	8 %	10 %	13 %	16 %	21 %	77 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	4 %	7 %	9 %	12 %	14 %	18 %	22 %	28 %	73 %	$\mathrm{AC}_{\mathrm{step}_4}$
	1 %	6 %	9 %	12 %	15 %	18 %	22 %	26 %	33 %	84 %	$\mathrm{AC}_{\mathrm{step}_5}$
3	0 %	0 %	0 %	0 %	0 %	3 %	5 %	9 %	14 %	69 %	LS
	0 %	0 %	0 %	0 %	2 %	3 %	4 %	5 %	8 %	46 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	2 %	4 %	4 %	6 %	8 %	10 %	13 %	52 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	1 %	4 %	5 %	7 %	8 %	11 %	13 %	18 %	60 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	3 %	5 %	7 %	9 %	11 %	13 %	17 %	22 %	67 %	$\mathrm{AC}_{\mathrm{step}_5}$
4	0 %	0 %	0 %	0 %	1 %	3 %	6 %	9 %	15 %	75 %	LS
	0 %	0 %	0 %	0 %	1 %	2 %	3 %	4 %	5 %	32 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	1 %	2 %	3 %	4 %	5 %	7 %	9 %	47 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	1 %	3 %	4 %	5 %	6 %	7 %	10 %	13 %	46 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	3 %	4 %	5 %	7 %	8 %	10 %	12 %	16 %	52 %	$\mathrm{AC}_{\mathrm{step}_5}$
5	0 %	0 %	0 %	0 %	1 %	4 %	6 %	10 %	15 %	69 %	LS
	0 %	0 %	0 %	0 %	1 %	2 %	2 %	3 %	4 %	49 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	0 %	2 %	2 %	3 %	4 %	5 %	7 %	49 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	0 %	2 %	3 %	4 %	5 %	6 %	7 %	10 %	49 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	2 %	3 %	4 %	5 %	6 %	8 %	10 %	13 %	55 %	$\mathrm{AC}_{\mathrm{step}_5}$
7	0 %	0 %	0 %	0 %	0 %	0 %	0 %	4 %	11 %	63 %	LS
	0 %	0 %	0 %	0 %	3 %	6 %	8 %	11 %	15 %	76 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	3 %	6 %	9 %	11 %	14 %	18 %	25 %	82 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	3 %	7 %	10 %	13 %	16 %	20 %	25 %	33 %	108 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	6 %	10 %	13 %	16 %	20 %	25 %	30 %	40 %	122 %	$\mathrm{AC}_{\mathrm{step}_5}$

Table IV: Cost differences Δ (part 2 of 2).

	Percentile										
Scenario	**10 %**	**20 %**	**30 %**	**40 %**	**50 %**	**60 %**	**70 %**	**80 %**	**90 %**	**100 %**	***R* by**
8	0 %	0 %	0 %	0 %	0 %	0 %	2 %	6 %	12 %	75 %	LS
	0 %	0 %	0 %	0 %	2 %	3 %	4 %	6 %	8 %	59 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	1 %	3 %	5 %	6 %	8 %	10 %	14 %	60 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	2 %	4 %	5 %	7 %	9 %	11 %	15 %	20 %	74 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	3 %	5 %	7 %	9 %	12 %	15 %	19 %	25 %	92 %	$\mathrm{AC}_{\mathrm{step}_5}$
9	0 %	0 %	0 %	0 %	0 %	0 %	2 %	6 %	12 %	82 %	LS
	0 %	0 %	0 %	0 %	2 %	2 %	3 %	4 %	5 %	51 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	1 %	2 %	3 %	4 %	5 %	7 %	10 %	58 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	1 %	3 %	4 %	5 %	6 %	8 %	10 %	14 %	80 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	2 %	3 %	5 %	7 %	8 %	10 %	13 %	18 %	84 %	$\mathrm{AC}_{\mathrm{step}_5}$
10	0 %	0 %	0 %	0 %	0 %	0 %	3 %	6 %	13 %	95 %	LS
	0 %	0 %	0 %	0 %	1 %	2 %	2 %	3 %	4 %	58 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	1 %	2 %	2 %	3 %	4 %	5 %	8 %	58 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	0 %	2 %	3 %	4 %	5 %	6 %	8 %	11 %	60 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	2 %	3 %	4 %	5 %	6 %	8 %	10 %	14 %	68 %	$\mathrm{AC}_{\mathrm{step}_5}$
13	0 %	0 %	0 %	0 %	0 %	0 %	1 %	7 %	15 %	104 %	LS
	0 %	0 %	0 %	0 %	2 %	3 %	4 %	6 %	8 %	68 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	2 %	3 %	5 %	6 %	8 %	10 %	15 %	83 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	1 %	4 %	5 %	7 %	9 %	11 %	15 %	22 %	89 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	3 %	5 %	7 %	9 %	12 %	16 %	20 %	28 %	103 %	$\mathrm{AC}_{\mathrm{step}_5}$
14	0 %	0 %	0 %	0 %	0 %	0 %	2 %	7 %	15 %	144 %	LS
	0 %	0 %	0 %	0 %	1 %	2 %	3 %	4 %	5 %	46 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	1 %	2 %	3 %	4 %	5 %	7 %	10 %	65 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	0 %	2 %	4 %	5 %	6 %	8 %	10 %	14 %	71 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	2 %	3 %	5 %	6 %	8 %	10 %	13 %	19 %	81 %	$\mathrm{AC}_{\mathrm{step}_5}$
15	0 %	0 %	0 %	0 %	0 %	0 %	2 %	7 %	15 %	181 %	LS
	0 %	0 %	0 %	0 %	1 %	2 %	2 %	3 %	4 %	59 %	$\mathrm{AC}_{\mathrm{step}_2}$
	0 %	0 %	1 %	2 %	2 %	3 %	4 %	5 %	7 %	73 %	$\mathrm{AC}_{\mathrm{step}_3}$
	0 %	0 %	2 %	3 %	3 %	4 %	6 %	8 %	11 %	68 %	$\mathrm{AC}_{\mathrm{step}_4}$
	0 %	1 %	2 %	4 %	5 %	6 %	8 %	10 %	15 %	70 %	$\mathrm{AC}_{\mathrm{step}_5}$

The results in Table IV show that $\mathrm{AC}_{\mathrm{step}_{2-5}}$ is not able to reach $C_{t,AC_{\mathrm{step}_1}}$ in most cases and also that this inability rapidly increases and cost differences grow with increasing level of *Q*, *R* discretization. Furthermore, the results in Table IV show that in most simulated scenarios LS outperforms $\mathrm{AC}_{\mathrm{step}_{2-5}}$ in term of the number of time series for which $C_{t,AC_{\mathrm{step}_1}}$ are reached. For example, in the scenario 15, LS reaches the best possible holding and ordering costs in at least 60 % of simulated time series while $\mathrm{AC}_{\mathrm{step}_{2-5}}$ performs equally only in 10 to 40 % cases. However, in all simulated scenarios for a certain number of time series $\mathrm{AC}_{\mathrm{step}_{2-5}}$ performs better than LS in term of reached Δ. If we go back for example to scenario 15, 80 % percentile of cost differences for LS is equal to 7 % while the same percentile is equal to 3 % in case of $\mathrm{AC}_{\mathrm{step}_2}$ and 5 % in case of $\mathrm{AC}_{\mathrm{step}_3}$.

We examine this pattern closely and for each simulated scenario we determine number of time series where $C_{t,AC_{\text{step}_{2-5}}} < C_{t,LS}$ (see Table V).

Table V: Number of time series where $\boldsymbol{C_{t,AC_{\text{step}_{2-5}}} < C_{t,LS}}$.

Scenario	$\mathbf{AC_{step_2}}$	$\mathbf{AC_{step_3}}$	$\mathbf{AC_{step_4}}$	$\mathbf{AC_{step_5}}$
2	26 %	17 %	12 %	9 %
3	38 %	29 %	23 %	18 %
4	43 %	35 %	30 %	25 %
5	48 %	42 %	36 %	32 %
7	17 %	12 %	8 %	6 %
8	25 %	20 %	15 %	13 %
9	29 %	24 %	20 %	17 %
10	31 %	27 %	23 %	21 %
13	26 %	21 %	17 %	15 %
14	29 %	25 %	22 %	19 %
15	30 %	27 %	25 %	22 %

The results in Table V show that LS tends to perform better than $\text{AC}_{\text{step}_{2-5}}$ when the intermittency of demand increases (see e.g. Table V, $\text{AC}_{\text{step}_2}$, scenarios 5; 10 and 15). On the other hand, for a given number of zero demand periods (i.e. the level of demand intermittency) the performance of $\text{AC}_{\text{step}_{2-5}}$seems to improve with growing variability of non-zero demand (see e.g. Table V, $\text{AC}_{\text{step}_2}$, scenarios 2-5).

Together with $C_{t,AC_{\text{step}_{2-5}}}$we also record the time spent on simulation experiments for each simulated arrangement of a scenario. These consumptions and also the computational times of LS and $\text{AC}_{\text{step}_1}$coming from [30] are summarized in Table VI.

Table VI: Consumption of computational time [min].

Scenario / *R* by	$\mathbf{AC_{step_1}}$	**LS**	$\mathbf{AC_{step_2}}$	$\mathbf{AC_{step_3}}$	$\mathbf{AC_{step_4}}$	$\mathbf{AC_{step_5}}$
2	110	38	28	13	8	6
3	397	62	103	47	28	19
4	889	100	227	103	59	38
5	1 548	150	418	181	103	68
7	45	37	12	6	4	4
8	163	53	42	20	12	8
9	356	74	91	42	24	16
10	622	115	159	71	41	27
13	59	41	16	8	5	4
14	129	54	34	16	10	7
15	226	74	59	27	16	11

It can be seen in Table VI that with increasing discretization of *Q*, *R* the consumption of computational time in $\text{AC}_{\text{step}_{2-5}}$ rapidly decreases and for most of the simulated arrangements of scenarios it is far more favourable than the consumption of computational time of LS.

As $\text{AC}_{\text{step}_{2-5}}$outperforms LS in term of C_t relatively often and its time consumption is significantly lower it makes sense to combine these two different strategies together to improve the performance of LS. To demonstrate this potential we select e.g. scenario 5 and for each simulated arrangement we instead of $C_{t,AC_{\text{step}_i}}$ use $min\left(C_{t,AC_{\text{step}_i}}, C_{t,LS}\right)$ in calculation of Δ according to Eq. (6). 1 to 100 % percentiles of these modified Δ values are summarized in Table VII together with LS Δ percentiles coming from Table IV and with the total computational time spent on LS + $\text{AC}_{\text{step}_i}$ (i.e. TCT) coming from Table VI.

Table VII: Modified cost differences Δ based on $min\left(C_{t,AC_{\text{step}_i}}, C_{t,LS}\right)$.

Percentile											
10 %	**20 %**	**30 %**	**40 %**	**50 %**	**60 %**	**70 %**	**80 %**	**90 %**	**100 %**	***R* by**	***TCT* [min]**
0 %	0 %	0 %	0 %	1 %	4 %	6 %	10 %	15 %	69 %	LS	150
0 %	0 %	0 %	0 %	0 %	0 %	0 %	2 %	3 %	49 %	$min\left(C_{t,AC_{\text{step}_2}}, C_{t,LS}\right)$	568
0 %	0 %	0 %	0 %	0 %	0 %	2 %	3 %	5 %	49 %	$min\left(C_{t,AC_{\text{step}_3}}, C_{t,LS}\right)$	331
0 %	0 %	0 %	0 %	0 %	1 %	3 %	4 %	6 %	49 %	$min\left(C_{t,AC_{\text{step}_4}}, C_{t,LS}\right)$	253
0 %	0 %	0 %	0 %	0 %	2 %	3 %	5 %	8 %	49 %	$min\left(C_{t,AC_{\text{step}_5}}, C_{t,LS}\right)$	218

The results in Table VII prove that combining LS together with $\text{AC}_{\text{step}_{2-5}}$ brings significant improvements in C_t. While the individual application of LS in scenario 5 leads to the minimal holding and ordering costs in up to 40 % of simulated time series and for another 50 % of time series there is a maximal difference in these costs up to 15 % a combination of LS and for example $\text{AC}_{\text{step}_1}$increases this number up to 70 % and another 20 % of time series differ in the minimal possible holding and ordering costs just up to 3 %. There is also a decrease in the maximal Δ for an item from 69 % to 49 %. Of course this ability of LS + $\text{AC}_{\text{step}_i}$is paid with a higher computational time compared to the individual LS application but when confronted to $\text{AC}_{\text{step}_1}$(i.e. 1548 minutes according to Table VI, scenario 5) *TCT* savings ranges from 63 to 86 %.

3. CONCLUSION

In this paper we propose an alternative combinatorial strategy of reducing searched solution space in intermittent demand stock management based on the past stock movement simulation. The combinatorial strategy involves an adjustable level of the discretization of control variables that are used within a selected inventory control policy. We combine this new strategy with the local search employing linear regression and bootstrapping to bound the reorder point and simulate (Q, R) inventory control policy using randomly generated data. The data is characteristic with an increasing intermittency and a non-zero demand variability. The outputs from simulation experiments show that combining these two different strategies aimed at reducing searched solution space brings a significant improvement in the trade-off among the minimal holding and ordering costs, required service level and the consumption of the computational time making the past stock movement simulation to be more applicable in extensive real life tasks dealing with sporadic demand stock management. As both strategies as well as the past stock movement simulation are easy to programme and update they can become a serious rival to the traditionally used parametric forecasting approaches that perceive a demand forecasting and an inventory control to be two separate stages [16].

In our future work we are going to focus on a further evolution of the simulation both in term of computational time savings and the minimization of inventory costs. The findings coming from [30] show a weak point of the local search to consist in an insufficient underestimating of reorder point by linear regression especially for time series with a relatively low intermittency. Thus, we focus either on a replacement of linear regression with a more suitable approach or on upgrades in the safety stock calculation. We also see the great potential in incorporating demand data aggregation into the simulation. The data aggregation together with an adoption of demand classification schemes belong to strategies improving forecasting performance of parametric approaches through a reduction of a demand variability and a number of zero demand periods [35]. Beside that our potential lies in additional savings of computational time through a shortening time series to be simulated as well as through a lead time decrease affecting the duration of sampling in bootstrapping that is used in the local search to reliably overestimate the reorder point. There is also a space to perform a deeper sensitivity analysis concerning how for example different lead times influence the trade-off between holding and ordering costs, fill rate and the computational time.

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